

# Heat flow and heat production from the Malingsbo granite, central Sweden

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D. Malmqvist, S. Å. Larson, O. Landström, and G. Lind 1983 03 25: Heat flow and heat production from the Malingsbo granite, central Sweden. *Bulletin of the Geological Institutions of the University of Uppsala*, N.S., Vol. 9, pp. 137—152. Uppsala. ISSN 0302-2749.

Heat flow and heat production have been calculated for a Proterozoic granite area in the Bergslagen district, central Sweden. Methods used have been temperature measurements mostly in shallow boreholes, in situ measurements of heat conductivity, laboratory analyses by INAA and DNA of U, Th, and K, and natural  $\gamma$ -logging in boreholes. The mean for heat flows measured in boreholes within the Malingsbo granite is  $76 \text{ mWm}^{-2}$  and the mean for heat production from 67 samples of the granite is  $7,1 \mu\text{Wm}^{-3}$ . Corrections for climate effects on temperature measurements in shallow boreholes are suggested in order to get better estimates of the temperature at great depth. By using the relation between heat flow and heat production the thickness of the Malingsbo granite is estimated to be at least 2 km.

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## Introduction

This paper summarizes results from an investigation on heat flow and heat production from a Proterozoic granite in the Fagersta—Lindesberg—Kopparberg area, central Sweden (Fig. 1).

Magnusson (1940) recognized three texturally different types of granite in the investigated area. They were called Malingsbo-, Enkullen-, and Fellingsbro granite respectively. The red to grey, me-

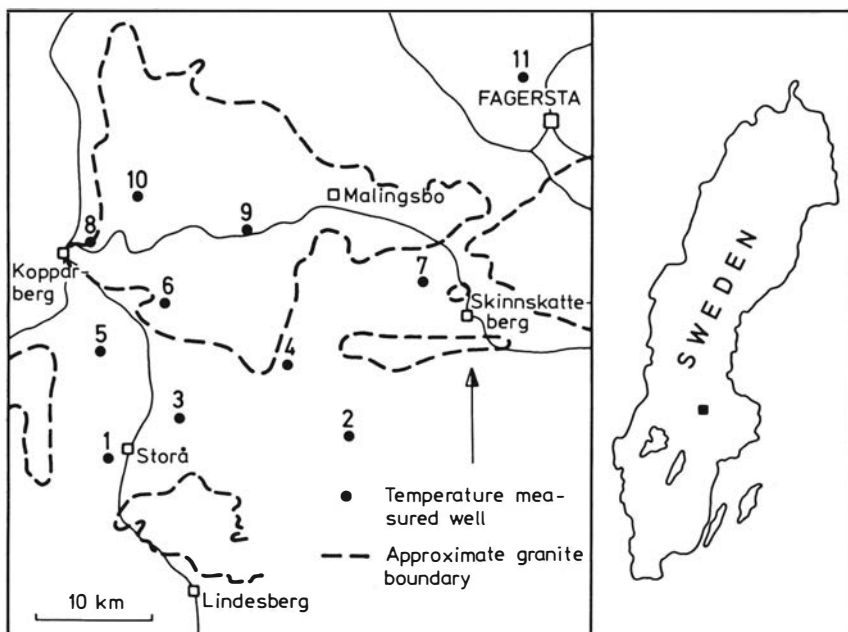


Fig. 1. Sites for temperature measured wells in the Malingsbo granite area. Boreholes 6 and 10 were especially drilled for heat flow measurement purpose.

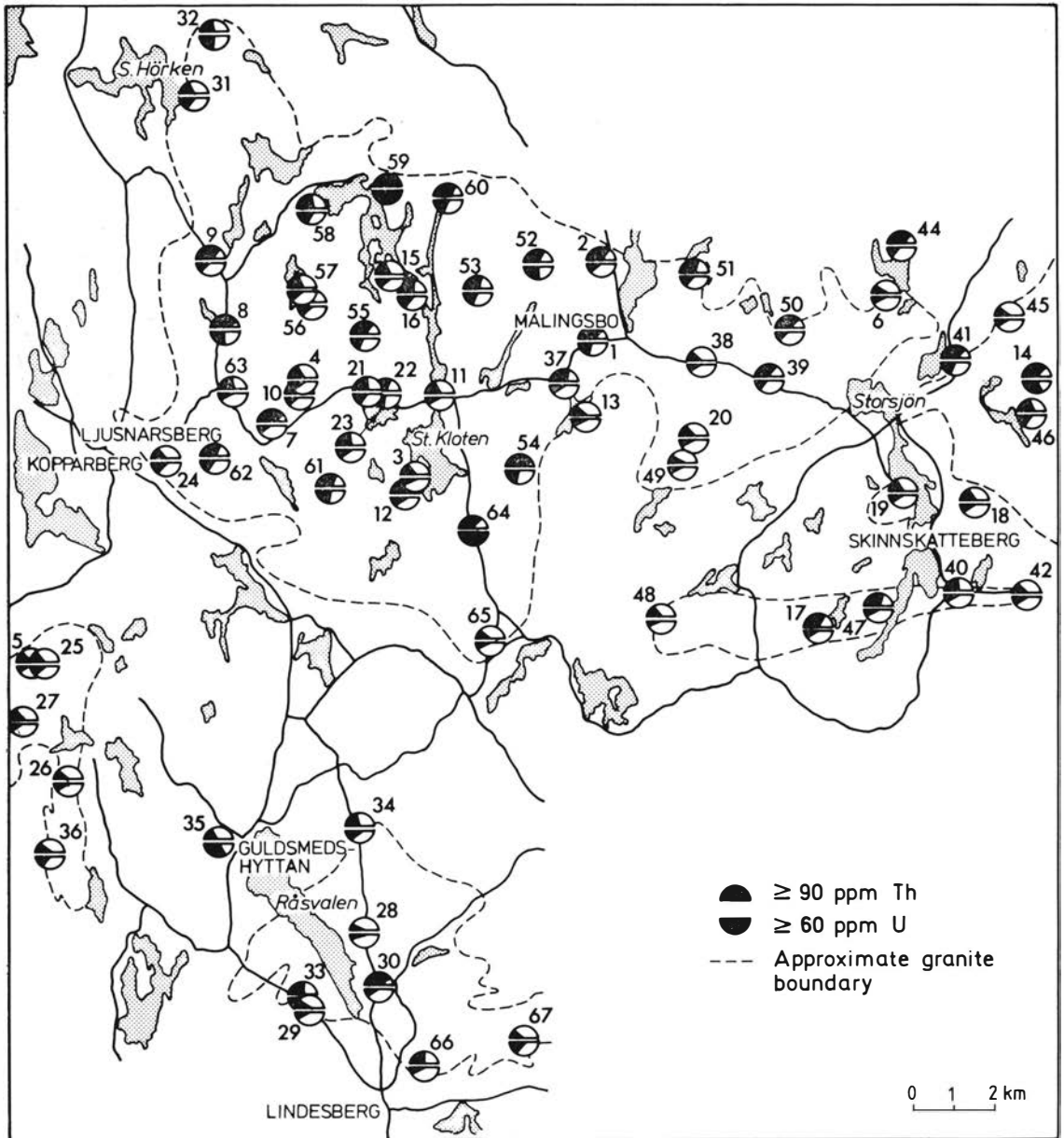


Fig. 2. Sites, as well as U and Th contents, for analysed samples from the Malingsbo granite area. Granite boundaries have been taken from different preliminary maps made by I. Lundström and M. Ambros at the Geological Survey of Sweden, and H. Koark and B. Collini, Institute of Geology, University of Uppsala.

dium, and even grained Malingsbo granite dominates the investigated area. In the field it is evident that this granite is intimately mixed with supracrustals; mainly fine grained meta-volcanics collectively called leptites (Magnusson, 1940). The areal distribution of granites *sensu stricto* is thus hard to estimate.

The temperatures were measured in available

wells including two vertical boreholes especially drilled for the purpose (Fig. 1). From these two boreholes cores were taken with a length of 150 metres each. The cores were sampled and mapped and the boreholes were investigated with a gamma ray spectrometer to find out the natural gamma activity along the boreholes. Surface samples were analysed for U, Th, and K.

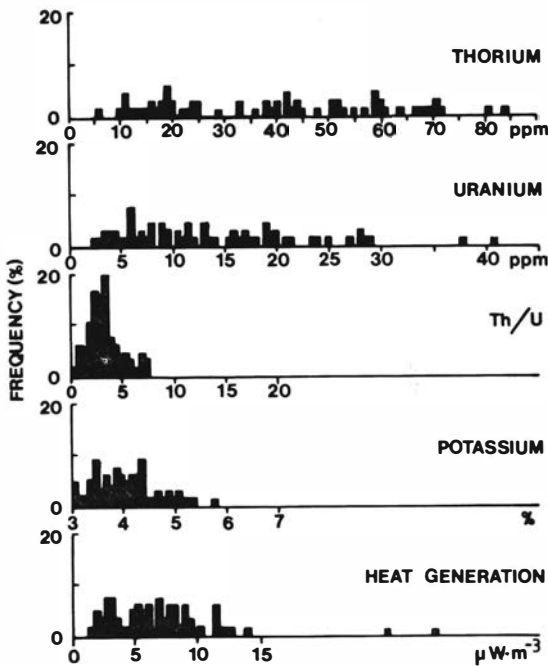


Fig. 3. Histograms showing the distribution of heat generating elements and heat production for 67 samples of Malingsbo granite.

### Heat production

Heat production was calculated from the U, Th, and K content (according to the formula given by Rybach 1973 in 67 samples taken from outcrops within the Malingsbo granite area (Fig. 2). Heat production was also calculated from analyses of sections from the two cores mentioned above. Th and K were analysed by instrumental neutron activation analyses (INAA) and U by delayed neutron activation (DNA). Fig. 3 shows the results of the analyses as well as calculated heat productions. Mean values for the 67 samples collected on the surface were 38,6 ppm Th, 15,7 ppm U, and 4,0 % K. The Th/U ratio was 3,2 and the heat production 7,1  $\mu\text{W}\cdot\text{m}^{-3}$ . The Malingsbo granite type (Magnusson 1940) shows a higher mean heat production (8,4  $\mu\text{W}\cdot\text{m}^{-3}$ ) than

texturally different granite types. It is also evident that the area west of Malingsbo village has an especially high heat production.

The mean heat production from the cores from boreholes number 6 and 10 (Fig. 1) is 7,1 and 5,3  $\mu\text{W}\cdot\text{m}^{-3}$  respectively. The lower value is explained by a relatively large leptite content.

### Geothermal parameters

Heat conductivity, heat diffusivity and heat capacity have been measured using the in situ method described by Landström et al. (1980). However, a phase displacement correction has been introduced (due to a delay in the heat flow in the clay used as filling between heating rod and the rock) in order to modify the calculations. Thus the formula is now written:

$$V = \frac{Q}{4\pi K} E_1(r^2/4\kappa(t+\tau)), \tag{1}$$

where

- $E_1(x) = -E_i(-x)$ , exponential integral
- $V$  = thermistor temperature
- $Q$  = effect/length
- $K$  = conductivity
- $\kappa$  = diffusivity
- $r$  = distance from the center of the heating rod to the thermistor
- $\tau$  = phase displacement
- $t$  = heating time.

For numerical calculations the following expression is used:

$$V = \frac{Q}{4\pi K} \left( \ln \frac{1}{\beta} - \gamma - \sum_{n=1}^n (-1)^n \frac{\beta^n}{n \cdot n!} \right) \tag{2}$$

where

$$\beta = \frac{r^2}{4\kappa(t+\tau)}$$

$\gamma$  = the Euler constant.

Calculations according to Horai (1971) for heat conductivity have been carried out as well as some divided bar measurements. Results are shown in Table 1. As can be seen, there is a good correlation

Table 1. Thermal parameters for granites from the Malingsbo area.

K(Wm <sup>-1</sup> °C <sup>-1</sup> )	$\kappa(\text{mm}^2 \text{s}^{-1})$	c(J kg <sup>-1</sup> °C <sup>-1</sup> )	N	Reference	Method
3,56 1)	1,75 2)	770 1)	?	Pratt et al. 1977	"Divided bar"
3,49 3)	—	—	6	This paper	" "
3,50 3)	—	—	5	" "	Modal composition
3,54	1,71	787	28	" "	In situ
Mean: 3,52	1,73	778			

1) Extrapolated to 10°C from the determined values using the determined thermal coefficients.

2) Calculated using  $\rho = 2636 \text{ kg m}^{-3}$ .

3) Laboratory temperature is considered to be 22°C.

of values from the different methods. Accordingly the heat conductivity is a little more than  $3,50 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ , the heat diffusivity is  $1,73 \text{ mm}^2\text{s}^{-1}$  and the heat capacity is  $778 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$ .

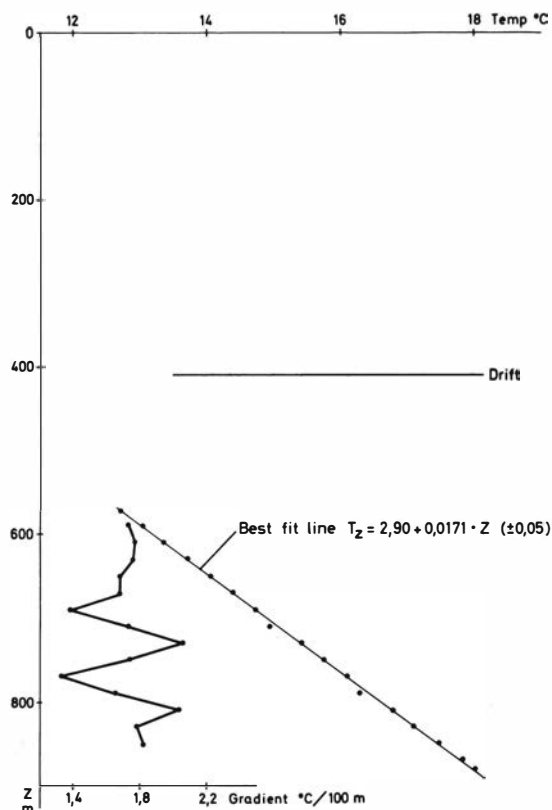
Heat conductivity for leptite have been measured by Hasselström (1972) using the divided bar method. The mean of 220 samples collected from two regions was  $3,36 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ . The mean of 120 samples from Stråssa was  $3,53 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ .

### Temperature measurements and climatic effects

Measured boreholes, geographical coordinates, and height above sea level are shown in Table 2. Only four of the boreholes are situated within the Malingsbo granite massif itself. Depth-temperature

Table 2. Geographical coordinates and altitudes at the boreholes in and around the Malingsbo area.

No.	Site	Lat.	Long.	H.a.s. (m)
1	Stripa	59°42',1	15°5',1	290
2	Grimso	59°44',7	15°28',4	110
3	Stråssa	59°45',0	15°12',4	229
4	Gammelbo	59°47',2	15°27',4	120
5	Rällsö	59°47',7	15°5',6	120
6	Malingsbo II	59°51',7	15°37',1	170
7	Sandvretsgruvan	59°53',2	15°8',5	295
8	Kopparberg	59°33',3	15°2',8	242
9	Kloten	59°53',9	15°18',2	272
10	Malingsbo I	59°55',3	15°8',3	340
11	Rudgruvan	60°1',2	15°45',9	320



Figs. 4 to 14. Temperature — depth plots for boreholes nos. 1—11. In most figures are also the temperature gradients plotted.

Fig. 4. Borehole no. 1. Original gradients ( $\Delta T/\Delta Z$ ;  $\Delta Z = 30 \text{ m}$ ).

plots are shown in Figs. 4—14. Differences in heat conductivity for different rocks cannot, except in a few cases, be traced in the depth-temperature plots.

In the boreholes drilled within mines (nos. 1, 3, 7, and 11) the measurements have been made down to depths of 350 to 850 metres; whereas drill-depth in other boreholes is cut 170 m or less. When evaluating the results it is therefore highly desirable to find a suitable technique for extrapolation in order that the temperature gradients calculated from temperature measurements and the heat flows are of similar value in varying geographical borehole positions and at varying

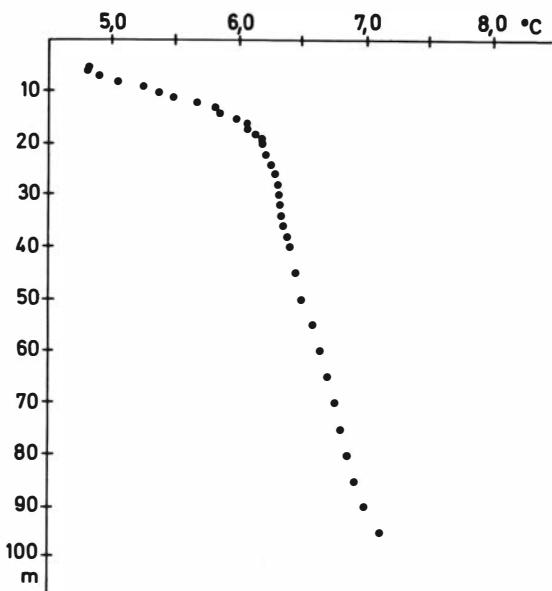


Fig. 5. Borehole no. 2.

depths. The corrections will vary accordingly as temperature measurements are made at relatively small or large depths.

In the first instance changes in temperature of the earth crust during earlier geological time-periods will not be taken into account. Thus, we regard the cause of disturbances in the temperature field of the upper crust of the earth mainly to be due to changes in earth surface temperatures in postglacial time. Consequently, a large part of the heat from the inner earth is used up in heating the upper layers of the crust.

Neither heat flow ( $H = K \cdot \partial T / \partial z = K \cdot G$ ) nor temperature gradients ( $G = \partial T / \partial z$ ) are therefore constant but vary as depth  $z$  and time  $t$ . Heat conductivity  $K$ , is also dependent on temperature (and hence on depth  $z$ ), but this fact will not be given further attention here.

The purpose of the extrapolation calculations is to receive corresponding values of temperature gradients as well as of heat flow values for the different boreholes at:

1. 800 m depth ( $G'$  and  $H'$ ). (Convention for temperature gradients and heat flow in deep boreholes in crystalline bedrock.)
2. 2000 m depth ( $G''$  and  $H''$ ). (Of interest in connection with extraction of geothermal energy).

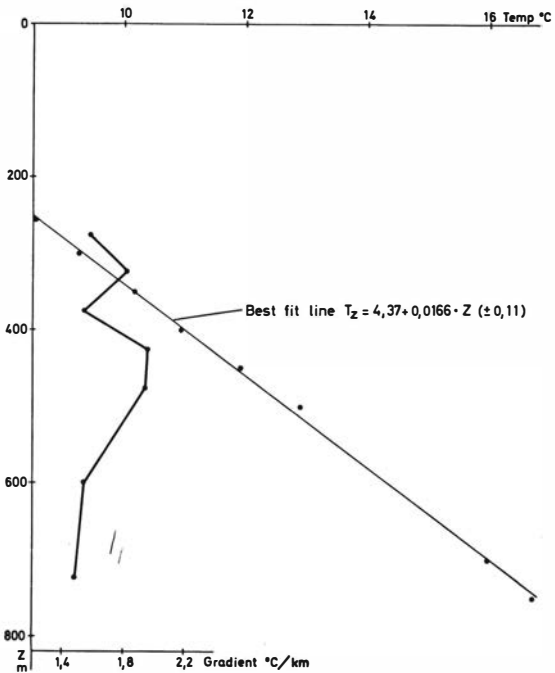


Fig. 6. Borehole no. 3. Original gradients. (n.b. for 'km' on lower graph read '100 m').

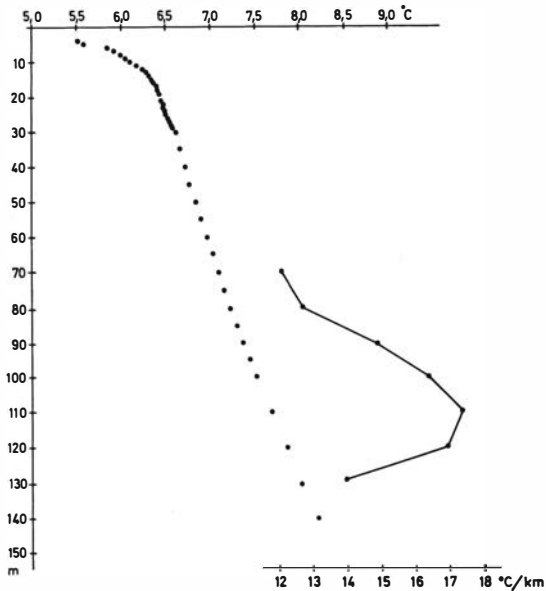


Fig. 7. Borehole no. 4. Original gradients.

$G$  and  $H$  have also been calculated. These values can be regarded as temperature gradients and heat flows in a temperature field in equilibrium at the end of the last glacial period.

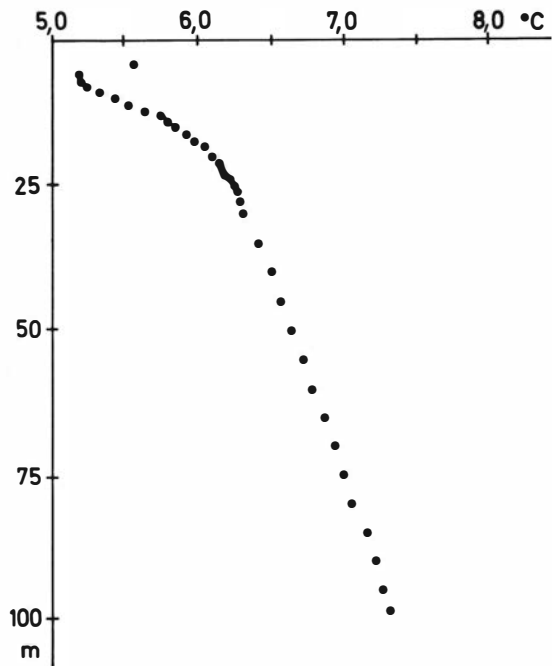


Fig. 8. Borehole no. 5.

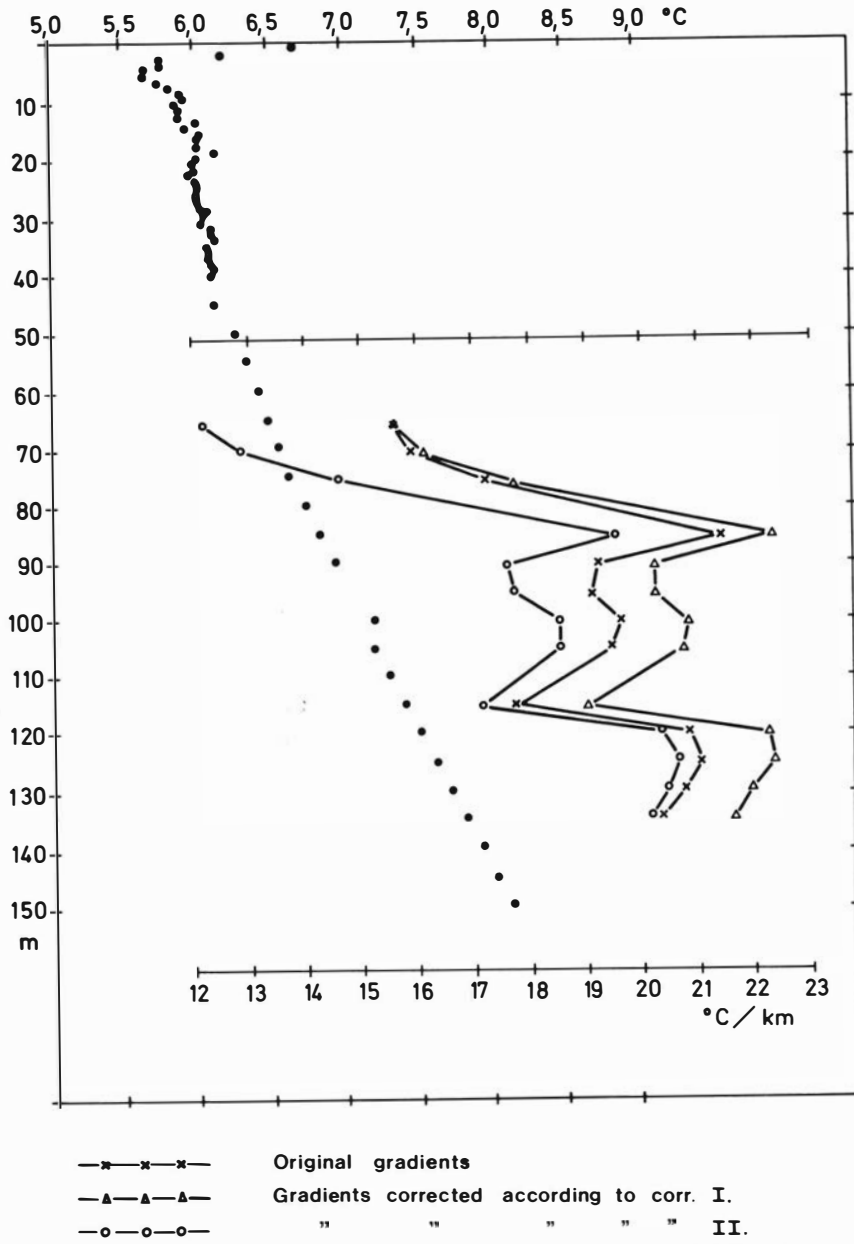


Fig. 9. Borehole no. 6. Corr. I — model Västerås; corr. II — model Falun.

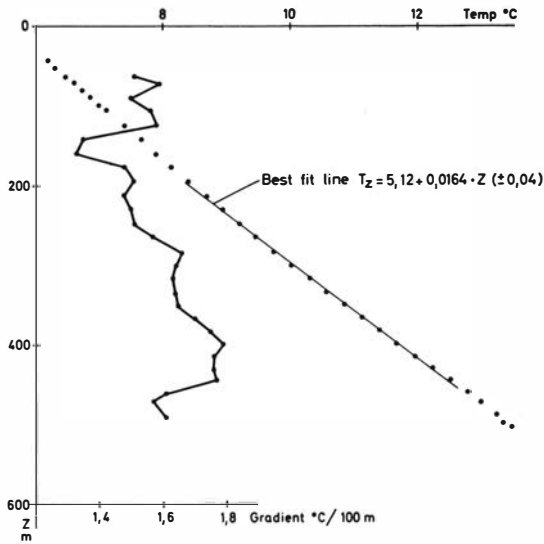


Fig. 10. Borehole no. 7. Original gradients.

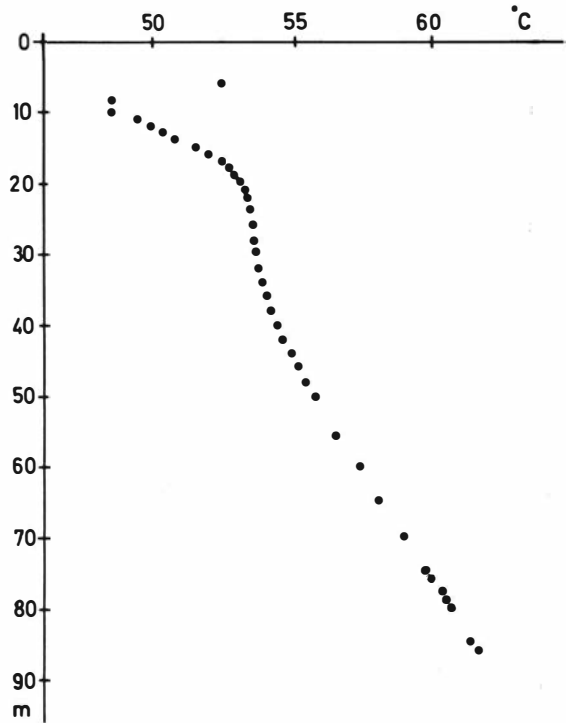


Fig. 12. Borehole no. 9.

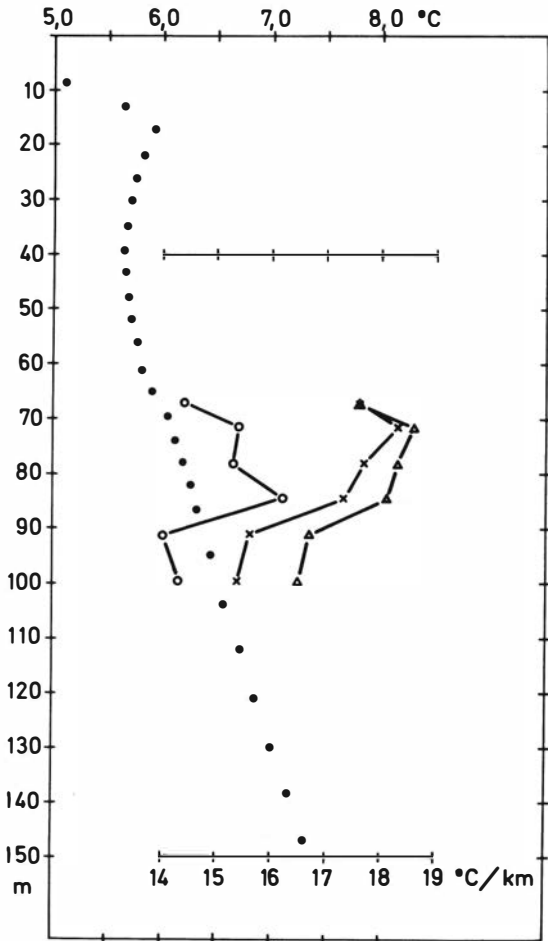


Fig. 11. Borehole no. 8. Concerning gradients see figure 9.

### Various types of climate influence

Changes in the post-glacial temperatures of the earth's surface can be regarded either as occurring in sudden steps,  $S$ , which took place  $t$  years ago or occurring in cyclic (sinus formed) harmonic changes during certain periods.

In the first case the temperature  $T$  at depth  $z$  can be calculated by using the formula:

$$T_z = T_0 + (T - T_0) \operatorname{erfc}(\gamma_1 z), \quad (3)$$

where

$T_0$  = Surface temperature before step  $S_1$ , ( $S_1 = T - T_0$ )

$T$  = Surface temperature after step  $S_1$

$\gamma_1$  = Damping factor,  $\frac{1}{2\sqrt{\kappa t_1}}$  (dimension:  $m^{-1}$ )

dependent on the diffusivity  $\kappa$  and the time  $t_1$ , from the step to present time.  $\operatorname{erfc}(x) =$  complementary function of error  $= (1 - \operatorname{erf}(x))$ , where  $\operatorname{erf}(x)$  is the error function.

The equation (3) is the solution of Fourier's one-dimensional partial heat-equation

$$\partial T / \partial t = \kappa \cdot \partial^2 T / \partial z^2,$$

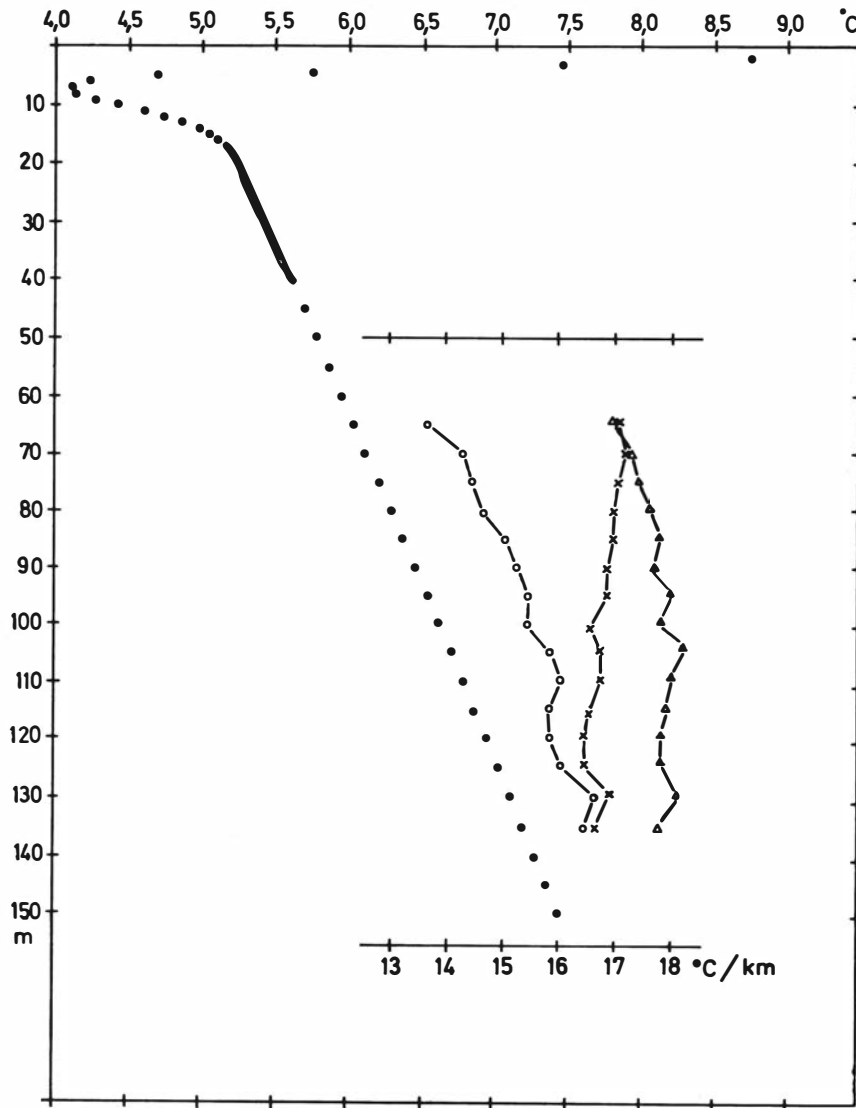


Fig. 13. Borehole no. 10. Concerning gradients see figure 9.

with the condition that

$$T = T_0 \text{ for } t > t_1 \text{ and } T = T \text{ for } t < t_1, \text{ if } z = 0$$

and

$$T_z = T \text{ if } t \rightarrow \infty$$

$$T_z = T_0 \text{ if } z \rightarrow \infty.$$

The above theoretical solution may be applied to a mathematical model corresponding to a homogeneous bedrock (homogeneous semi-space) if the surface temperature changes in steps.

An additional temperature component,  $G \cdot z$ , which is due to the constant heat flow from the mantle or lower crust, is added for the calculation of  $T_z$ . This component rises linearly with depth  $z$  and the coefficient  $G$  is equivalent to the steady state temperature gradient.

Finally we can make the assumption that the surface temperature at the end of the last glacial epoch was approximately  $0^\circ\text{C}$  — or insignificantly less owing to high water pressure at the bottom of the icecover — and that this temperature was sustained for thousands or even tens of



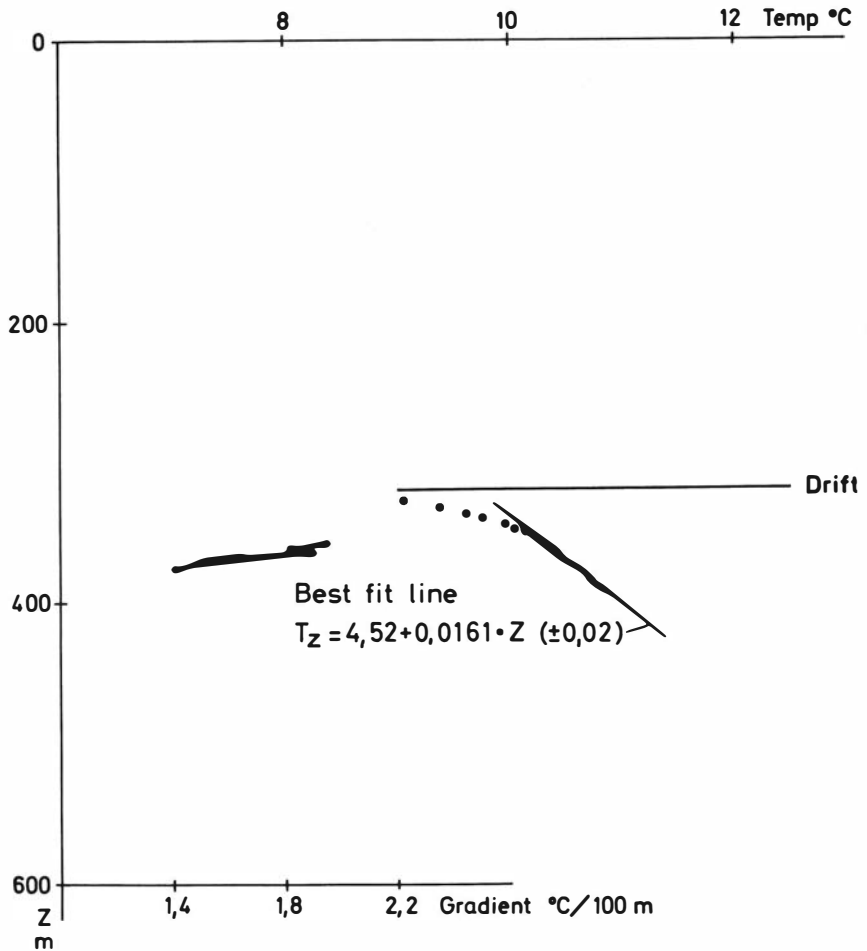


Fig. 14. Borehole no. 11. Original gradients.

thousands of years. Equation (3) may then be written:

$$T_z = G \cdot z + S_1 \cdot \operatorname{erfc}(\gamma_1 \cdot z). \tag{4}$$

Should a number of temperature steps,  $S_1, S_2, S_3, \dots, S_n$  since the end close of the glacial epoch have occurred  $t_1, t_2, t_3, \dots, t_n$  years ago, the following corollary applies:

$$T_z = G \cdot z + \sum_{n=1}^n S_n \cdot \operatorname{erfc}(\gamma_n \cdot z). \tag{5}$$

Thus the temperature gradient becomes:

$$\partial T_z / \partial z = G - \frac{2}{\sqrt{\pi}} \sum_{n=1}^n \gamma_n \cdot S_n \cdot \exp - (\gamma_n \cdot z)^2. \tag{6}$$

Since the end of the last glacial epoch in Sweden several temperature changes have occurred

at the surface of the earth. The first step,  $S_1$ , at the close of the glacial epoch, is the largest. Steps following  $S_1$  are significantly smaller. It is however necessary to consider at least one but sometimes two steps,  $S_2$  and  $S_3$ , when interpreting temperature — depth curves; not because of their greater amplitude compared to  $S_1$  but because they are closer in time to the present (= relatively high  $\gamma$ -values).

### Corrections for climate effects

The important climate effects we should try to make corrections for are as follows:

- a) Glacial; influence of Step  $S_1$ . (Noticeable down to depths of 2500 m.)

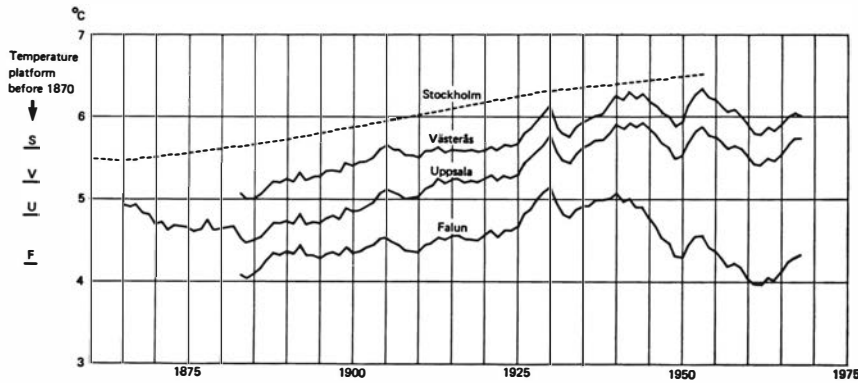


Fig. 15. Meteorological data on air temperatures for Stockholm, Uppsala, Västerås and Falun during the last century. The plotted temperatures are the average of mean annual temperatures in intervals of 30 (Stockholm) or 20 (Uppsala, Västerås, Falun) years.

- b) Secular; influence of Step  $S_2$ . (Noticeable down to depths of 250 m.)
- c) Decennial; influence of Step  $S_3$ . (Noticeable down to depths of 150 m.)
- d) Annual; influence of the annual temperature fluctuation. (Noticeable down to depths of 50 m.)

The temperature in the bedrock, which is dependent on annual temperature changes at the surface of the earth, varies like a damped sinus wave and is mathematically treated as such.

Measured temperatures down to depths of 50 m can be used for calculation of the diffusivity  $\kappa$  (Parasnis 1975; Landström et al. 1978), but we shall not penetrate more deeply into this matter in the following since  $\kappa$  may be regarded as known for the bedrock in the region. Thus temperatures in bore-holes at depths of less than 50 m will not be considered in this paper and corrections as of d) above will not be needed.

If the temperature at a given depth  $z_1$  is known and the temperature gradient,  $(\partial T/\partial z)_{z_1}$  plus the damping-factor,  $\gamma_1$ , are applied, we may use equations (5) and (6) to calculate both the magnitude of  $S_1$  and the temperature gradient  $G$  (in equilibrium). Thus

$$S_1 = \frac{T_{z_1} - z_1 \cdot \frac{\partial T_{z_1}}{\partial z}}{\operatorname{erfc}(\gamma_1 \cdot z_1) + \frac{2\gamma_1 z_1}{\sqrt{\pi}} \exp -(\gamma_1 \cdot z_1)^2} \quad (7)$$

and

$$G = \frac{T_{z_1} - S_1 \operatorname{erfc}(\gamma_1 \cdot z_1)}{z_1} \quad (8)$$

For the calculations listed in table no. 3 the damping-factor  $\gamma_1$  is put as  $= 7,3 \cdot 10^{-4} \text{m}^{-1}$  calculated from a  $t_1 = 8600$  years and a diffusivity  $= 1,72 \text{mm}^2/\text{s}$ .

Once  $S_1$  is determined then both  $T_z$  and  $\partial T_z/\partial z$  may be calculated for any desired depth. The magnitudes of temperature gradients and heat flows at certain other depths  $z_2$  are of special interest. These can be calculated from the values found at depth  $z_1$ , according to the formulae:

$$\frac{\partial T_{z_2}}{\partial z} = \frac{\partial T_{z_1}}{\partial z} + \frac{2\gamma_1 S_1}{\sqrt{\pi}} (\exp(\gamma_1 \cdot z_1)^2 \quad (9)$$

$$- \exp -(\gamma_1 \cdot z_2)^2)$$

or

$$\frac{\partial T_{z_2}}{\partial z} = \frac{\partial T_{z_1}}{\partial z} + f \cdot S_1, \quad (10)$$

$$\text{where } f = \frac{2\gamma_1}{\sqrt{\pi}} (\exp -(\gamma_1 \cdot z_1)^2 - \exp -(\gamma_1 \cdot z_2)^2).$$

### Calculations of temperature steps $S_2$ and $S_3$ .

The values for the annual temperature means (taken from the annual records of The Swedish Meteorological and Hydrological Institute) are shown in Figure 15 (calculated from the annual temperature means over a period of 20 years from 10 years earlier to 9 years later than the emplaced point). Records of annual temperature means are available from 1855 onwards (Uppsala) and from 1868 onwards (Västerås and Falun).

Temperatures in Stockholm seem to follow the same pattern as that of Uppsala and Västerås.

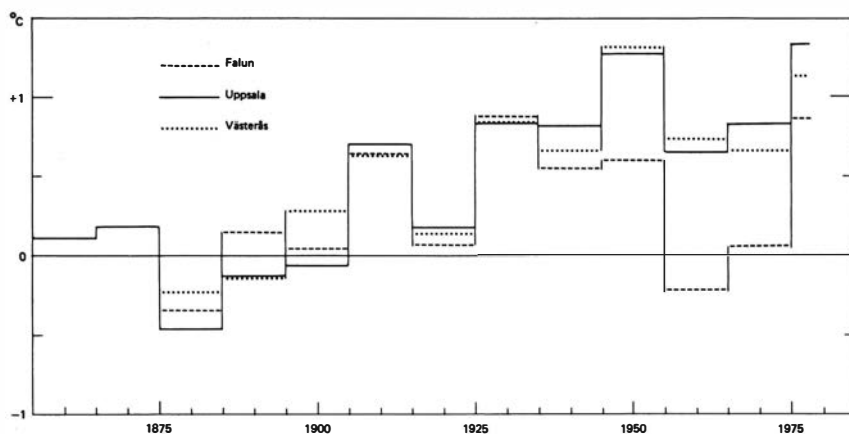


Fig. 16. Temperature steps between 10 year intervals (average of mean annual temperatures in the intervals 1855—1864 (step 1865), 1865—1874 (step 1875), 1875—1884 (step 1885) and so on). Last step is 1975.

(Running thirty-year annual temperature means for Stockholm are shown in Figure 15 as a broken line (Bolin 1968).) These means are based on calculated annual means 1845—1968. Records of annual temperature means for Stockholm have been made since 1756 and running thirty-year temperature means for Stockholm remained largely unchanged between 1771 and 1870. Hereinafter this level will be designated “ $0_1$ ”. For Stockholm is  $0_1 = 5,6^\circ\text{C}$ . If similar  $0_1$  levels existed at other points of observation we would have  $5,2^\circ\text{C}$  for Västerås,  $4,8^\circ\text{C}$  for Uppsala and  $4,2^\circ\text{C}$  for Falun. (This is on condition that the mean difference in annual mean temperature between Stockholm and other stations was the same before 1870 as between 1859 and 1925 (Wallén 1930).)

We can now calculate temperature and temperature gradients (eqs (5) and (6)) in the bedrock caused by steps (1855, 1865, 1875, 1885 etc of figure 16) which have occurred during the last century up to the beginning of 1978. Temperatures of figure 17 are those which lie above the  $0_1$  level and temperature gradients of figure 18 are changes in gradients caused by heating during the last century. The calculations are carried out using a diffusivity of  $1,8\text{mm}^2\text{s}^{-1}$ . Results would not differ significantly even if the diffusivity is varied (see figure 17).

It is now instructive to enquire whether or not we may replace all the steps determined (1855, 1865, 1875, etc.) with one single step,  $S_2$ . It turns out that such a course is possible in respect of Uppsala and Västerås where:

for Uppsala;

$$T_z = 4,80 + 0,85 \operatorname{erfc}(0,0084 z) \quad (11)$$

and for Västerås;

$$T_z = 5,20 + 0,81 \operatorname{erfc}(0,0083 z). \quad (12)$$

In the case of Falun it is necessary to use two steps,  $S_2$  and  $S_3$  (see Figure 19). Where  $S_2 = S_3$  we obtain:

$$T_z = 4,20 + 0,50 (\operatorname{erfc}(0,0078 z) - \operatorname{erfc}(0,0141 z)). \quad (13)$$

Formulae (11)—(13) make the best fit (least squares method) to temperatures obtained where all the steps (1855, 1865, 1875, etc.) are taken into consideration. Where  $60\text{ m} < z < 200\text{ m}$ , is the standard deviation for the temperatures calculated only  $0,0025 - 0,0035^\circ\text{C}$  irrespective of method of calculation.

The value of  $\gamma_2$  (corresponding to  $S_2$ ) as of equations (11) to (13) gives  $t_2$  values of 62,1 years for Uppsala, 64,7 years for Västerås, and 72,4 years for Falun (using  $\kappa = 1,8\text{mm}^2\text{s}^{-1}$ ). Thus, should but one step be assumed to have occurred then it will appear to have done so in 1916, 1913, and 1906 respectively. The value of  $\gamma_3$  (corresponding to  $S_3$ ) is calculated with the assumption that the cooling step took place 22 years earlier (1955/1956).

A study of data from the measured boreholes within the investigated area shows that the temperature changes are closely related to the Falun

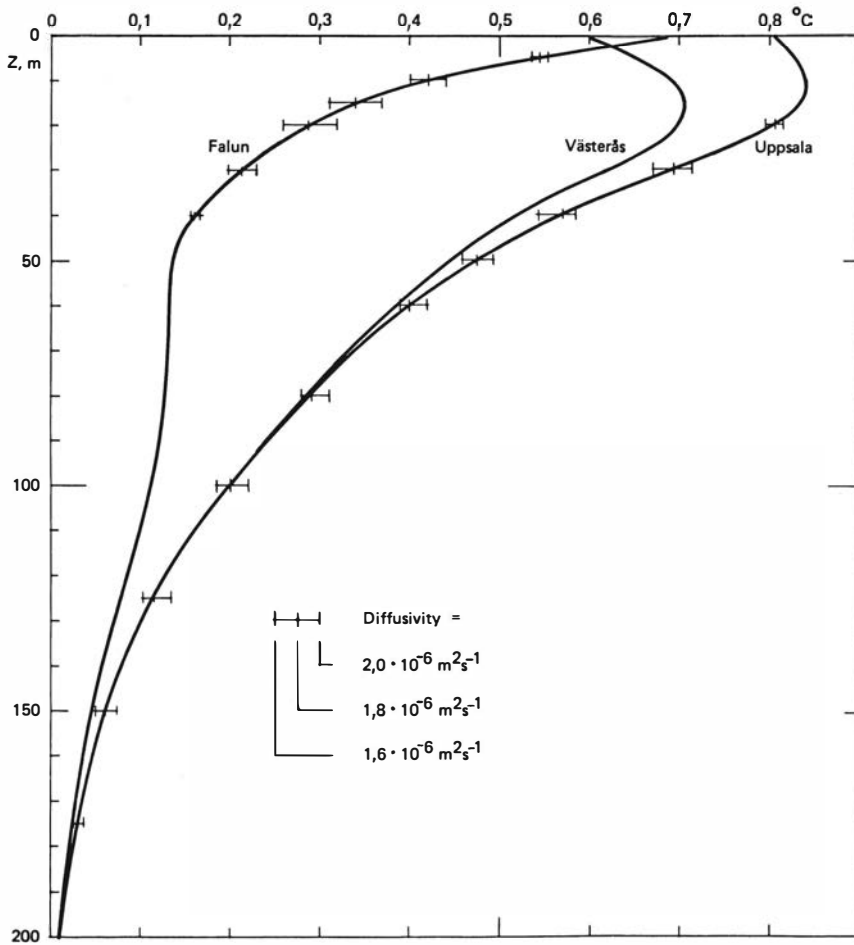


Fig. 17. Increased temperatures of the ground down to 200 m during the last century at the observation sites Uppsala, Västerås and Falun. The temperature increases above the 0<sub>1</sub>-level, depend on the climate amelioration in the last century. A diffusivity of 1,6—2,0 mm<sup>2</sup>s<sup>-1</sup> of the rock is supposed.

model and the corrections applied are calculated in accordance with this model (with one exception). The bedrock has been heated via step S<sub>2</sub> in accordance with the Falun model, but in contrast to the Västerås—Uppsala model the bedrock has been cooled down again owing to the negative step S<sub>3</sub> of 1955/56. Thus the heating caused by step S<sub>2</sub> is relatively insignificant at depths greater than 60 m. The correction for the heating is as follows for varying depths *z* (60 m < *z* < 200 m):

$$\text{Correction (Falun)} = -0,50 (\operatorname{erfc}(0,0078 z) - \operatorname{erfc}(0,0141 z)).$$

The corresponding correction according to the Västerås model is:

$$\text{Correction (Västerås)} = -0,81 \operatorname{erfc}(0,0083 z).$$

Step S<sub>2</sub> causes decrease of the temperature gradients in the upper parts of boreholes within the area (positive correction) as follows:

$$\begin{aligned} \text{Correction } \frac{\partial T_2}{\partial z} (\text{Falun}) &= \\ &= 0,004 \exp - (0,0078 z)^2 - 0,0079 \\ &\quad \exp - (0,0141 z)^2, \end{aligned}$$

$$\text{Correction } \frac{\partial T_2}{\partial z} (\text{Västerås}) = 0,0075 \exp - (0,0083 z)^2$$

as of equation (6).

Parasnis (1975) has elaborated an indirect method to establish the magnitude of the step S<sub>2</sub>. This method has been used on two of the bore-

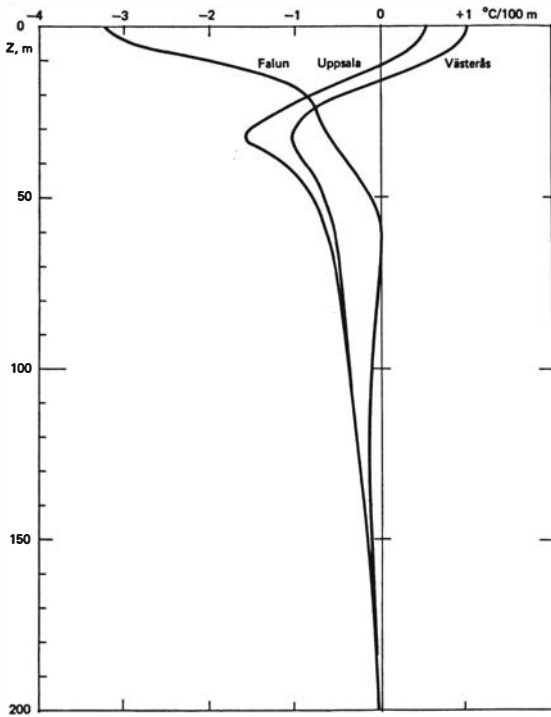


Fig. 18. Changes of the temperature gradients of the ground down to 200 m caused by the climatic amelioration in the last century.

holes and seems to give about the same results as the direct method described in the present paper.

### Interpretation of the results of temperature measurements

In summary of the results we may state that the post-glacial step  $S_1$  is approximately 3 to 6°C and the secular  $S_2$  is approximately 0,5 to 1,1°C. The decennial  $S_3$  appears to be of the same order of magnitude as is  $S_2$ .

As has been pointed out earlier extrapolations have been made according to the Falun model in the seven boreholes wherein temperature measurements have not been carried out at depths greater than 250 m. But as has been shown, the Falun model may not be applicable to all the boreholes, especially not for the boreholes lying in the southern part of the investigated area. If, however, the Västerås model is used temperature gradients and heat flows of such magnitudes are obtained that the readings seem less probable by comparison with the values obtained in other boreholes where temperatures have been measured at large depths. Therefore the Falun model, or a model approximating to the same, seems more representative for

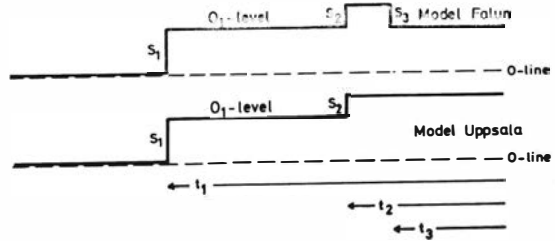


Fig. 19. Suggested steps  $S_1$ ,  $S_2$  and  $S_3$  for the models Falun and Uppsala (or Västerås which has the same character).

the area than does the Västerås model. We will probably not be able to get better values for  $S_2$  and  $S_3$  except through a better pattern of borehole measurements or through further studies of climatic changes over the last century for stations closer to the investigated area.

Values for  $S_1$ ,  $G'$ ,  $G''$ , and  $G$  for the four deeper boreholes are given in Table 3 as calculated according to formulae 7—10. As regards the other seven boreholes, the values  $S_1$ ,  $G'$ ,  $G''$ , and  $G$  are calculated in the same way but with corrections according to the Falun model. The values for surface temperatures shown in columns 5 and 6 in Table 3 are calculated according to the following formulae:

$$T_s = 0,66 (60 - \phi) - 0,0081 h + 5,85 \quad (14)$$

for the period 1931—1960

and

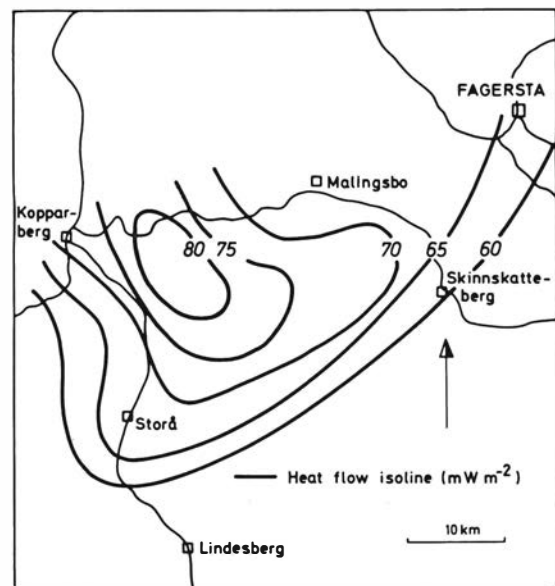


Fig. 20. Heat flow isolines of the investigated area.

Table 3. Calculated surface temperatures, temperature steps  $S_1$ , observed temperatures and gradients at some fixed depths and heat flows ( $G'$ ,  $G''$  and  $G$  resp.  $H'$ ,  $H''$  and  $H$ ) at 800 m, 2000 m and in "steady state".

Borehole no.	Rock type	$z_1$ m	Surface temp. 1931—60 °C	Surface temp. 1859— 1925 °C	$T_{z_1}$ °C	$\partial T_{z_1} / \partial z$ °C/km	$S_1$ °C	$G'$ °C/km	$H'$ mWm <sup>-2</sup>
1 Stripa	gr	710	3,70	3,55	15,04	17,1	3,18	17,2	61
2 Grimsö	l	75	5,13	4,71	6,70	12,4	5,77	13,8	46
3 Stråssa	l, gn	500	4,16	3,92	12,67	16,6	4,52	17,2	51
4 Gammelbo	gr, gre	100	5,02	4,61	7,33	16,2	5,71	17,5	62
5 Rällsö	l	75	5,01	4,61	6,99	13,8	5,96	15,2	51
6 Malingsbo II	gr, gn, gre	100	4,56	4,23	7,08	20,0	5,08	21,2	75
7 Sandvretsgruvan	l, sk, o	310	3,54	3,38	10,20	16,4	5,16	17,4	59
8 Kopparberg	gr, gre	87	3,96	3,73	6,20	17,3	4,70	18,4	64
9 Kloten	gr	70	3,72	3,53	5,90	15,8	4,79	16,9	60
10 Malingsbo I	gr, gn, gre	100	3,15	3,06	6,51	17,7	4,74	18,8	66
11 Rudgruvan	l, sk, o	387	3,24	3,12	10,75	16,1	4,59	16,9	57

gn = gneiss                    l = leptite  
 gr = granite                    o = ore/ore impregnation  
 gre = greenstone                sk = "skarn"

$$T_s = 0,72 (60 - \phi) - 0,066 h + 5,25 \quad (15)$$

for the period 1859—1925

where  $\phi$  is the latitude of the borehole and  $h$  its height above sea level (Eriksson & Malmqvist 1979).

As is evident from Table 3 the extrapolated surface temperatures in all the boreholes except no. 1, Stripa, are lower than the  $S_1$  values. They are, on the average, 1°C lower. This is quite a normal phenomenon and it probably depends on a higher summer inflow of heat through surface layers than the corresponding winter outflow. The differences seem also to be greatest in respect of boreholes situated in geographically high places, i.e. in localities where snow-cover has remained longest in the year.

In the sketch-map, figure 20, are isolines for heat flow shown. It is based on the heat flow values from the 11 boreholes examined. These values have been corrected in respect of climatic effects ( $G$ -values). The results confirm that the heat flows both within and outside the Malingsbo massif are 1,5—2 times higher than the values expected in Swedish bedrock. Within the Malingsbo massif itself the heat flow is even somewhat higher than in the surrounding anomalous areas. In the west, the south, and the southeast high horizontal heat flow gradients coincide largely with the boundaries of the Malingsbo massif. Whether or not this applies to the northern boundary is extremely uncertain inasmuch as we know that heat flows measured in boreholes 25 km NW of Malingsbo are almost as high as the highest within the Malingsbo massif. Thus it is evident that this area belongs to a region of

central Sweden which shows particularly high heat flows.

### The connection between heat flow and heat production

The mean heat flow measured in boreholes within the Malingsbo massif (boreholes nos. 6, 8, 9, 10) is 76 mWm<sup>-2</sup>. Heat flow from other boreholes show a mean of 67 mWm<sup>-2</sup>. We may therefore say that the granite pluton which has intruded into the country-rocks (leptites, gneisses, older granites) has contributed approximately 9 mWm<sup>-2</sup> to the heat flow.

As mentioned earlier the average heat production of the Malingsbo granite is 7,1  $\mu$ Wm<sup>-3</sup>. Also the country rocks show relatively high heat production values. According to Landström (1978) leptites from Stråssa have a heat production value of 2,7  $\mu$ Wm<sup>-3</sup>; for leptites from Blötberget heat production values are at 2,5  $\mu$ Wm<sup>-3</sup>, and for a Proterozoic granite from Risbergsfältet at 2,8  $\mu$ Wm<sup>-3</sup>. The heat production value for Stråssa may therefore be said to be quite representative of the surrounding bedrock of the Malingsbo granite massif.

The statement that the heat flow at the earth's surface is linearly related to the heat production of outcropping crystalline rocks has often been put forward. For a granite pluton we have:

$$Q = a + bP, \quad (16)$$

where

$Q$  = heat flow at the surface

$a$  = heat flow from the mantle and the lower crust

z<sub>1</sub>, extrapolated gra-

G''	H''	G	H
°C/km	mWm <sup>-2</sup>	°C/km	mWm <sup>-2</sup>
18,8	66	19,1	67
16,6	56	17,1	58
19,4	68	19,9	70
20,3	72	20,9	73
18,1	61	18,7	63
23,7	83	24,2	85
19,9	67	20,4	69
20,7	73	21,1	74
19,3	68	19,7	69
21,1	74	21,6	76
19,1	64	19,6	66

b = depth of the pluton

P = heat production from the pluton.

This simple and rather schematic relationship is quite perceptible in granite areas. This is because the depths (b) are mostly identical (6 to 8 km) and because the heat flow contribution from crustal layers beneath the pluton is small (but not insignificant). Cases equating with the phenomena could be said to belong to the "pluton group" ("Type I" according to Malmqvist 1978).

In crystalline bedrock with outcropping rocks of supracrustal origin a linear relationship between heat flow and heat production seems also to occur. However, the b-values appear to be twice as high as in other cases and to lie at between 13 to 16 km. On the other hand the a-value is somewhat lower, which may be explained by the much thinner layer of the lower crust than in the "type I" cases. Cases equating with the given phenomena have been called the "supracrustal group" (Type II).

In Figure 21 are drawn the straight lines A, B, C, D, and E from various Precambrian shield areas. These are lines best fitted to the empirically determined linear relationships between measured heat flows and heat production values within the areas delineated in the figure. Lines A and B belong to the "supracrustal group" and line C belongs to the "pluton group". The last named group is represented by granite plutons in the United States and in Australia. Line D emerges from a statistical calculation wherein all determinations of both heat flows and heat production values from Precambrian shield areas are included. Line E indicates a type of linear relationship which deviates from the other types

owing, probably, to an anomalously low heat production in the rocks of the crust.

The Malingsbo massif is obviously an interesting example of a combination of the types of linear relationship described above. We have here to do with a granite pluton of small thickness which has been intruded into a country rock within which the geothermal conditions coincide with those of the "supracrustal group". Stråssa (Figure 21) may be regarded as characteristic of the country rock according to line A. This is represented by the equation:

$$Q = 25 + 15,6 P \quad (17)$$

where

$$a = 25 \text{ mWm}^{-2}$$

$$b = 15,6 \text{ km.}$$

We now assume that we have a strongly heat-generating upper layer (granite) with a heat production  $P_2$  down to a given depth,  $b_2$ , plus an underlying layer of rock (identical to the country rock of the surface layer) reaching depth  $b_1$  and having a heat production  $P_1$  we may modify equation (16) as follows:

$$Q = a + (b_1 - b_2)P_1 + b_2P_2. \quad (18)$$

Where equation (18) is applied to the Malingsbo area, where  $Q = 76 \text{ mWm}^{-2}$ ,  $a = 25 \text{ mWm}^{-2}$ ,  $P_2 = 7,1 \text{ } \mu\text{Wm}^{-3}$ ,  $P_1 = 2,7 \text{ } \mu\text{Wm}^{-3}$ , and  $b_1 = 15,6 \text{ km}$ , the value of  $b_2$  will be 2,0 km.

In Figure 21 line (25 + 13,6 P), where  $0 < P < 2,7$ , corresponds to line A<sub>1</sub> and line (62 + 7,1 P), where  $0 < P < 7,1$  corresponds to line A<sub>2</sub>.

In accordance with this two-layers interpretation, the average thickness of the Malingsbo massif would thus be approximately 2 km. This value tallies quite well with that arrived at by Werner et al. (1977) from gravity measurements in the area.

This depth must, however, be regarded as a minimum value since it seems unlikely that the heat production of the granite would increase with depth. On the contrary, the heat production is mostly considered to decrease with depth. Then the average thickness of the Malingsbo granite ought to be greater than 2 000 metres.

Variable heat flow values registered within the massif probably reflect a variation in depth of the granite and there is reason to believe that the depth is greatest in the southern part of the granite area. According to equation (18) the calculated value of the heat flow is  $62 \text{ mWm}^{-2}$ . This value is considerably lower than the calculated average H''-value (at depth 2 km) this being  $74 \text{ mWm}^{-3}$  (Table 3). This latter value, however, is calculated on the assumption that the bedrock is

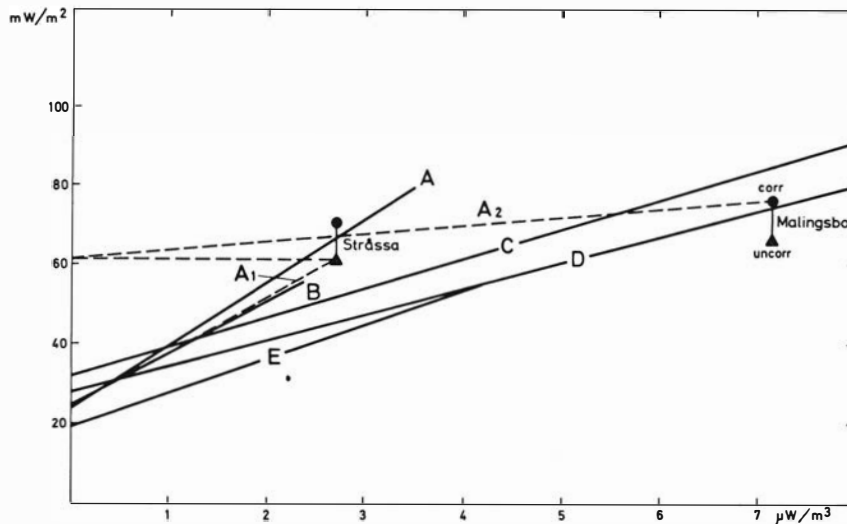


Fig. 21. Relationship between heat flow and heat production in Precambrian shield areas. Best fit lines concerning: A. Middle Sweden, B. Kapuskasing Area, Canada, C. Plutons in Eastern United States and Australia, D. All shields, E. Southwestern Norway. A<sub>1</sub> and A<sub>2</sub> represent relationships which explain heat flow from two heat-producing layers of the upper crust.

homogeneous and has been heated up after the close of the glacial epoch. The climatic effects of step S<sub>1</sub> could conceivably be somewhat different where the existence of two layers in the upper crust is assumed. The heat flow in the lower part of the granite ought therefore to lie between 62 mWm<sup>-2</sup> and 74 mWm<sup>-2</sup> (and probably closer to the latter value).

The temperature gradients G'' may, on the other hand, be higher than the values given in Table 3 owing to the fact that the heat conductivity is 5–6% lower and the temperature 40–50°C higher than in the surface layer. The temperature in the bedrock under the granite ought thus to be higher than corresponding temperatures at the same depth in the environs of the granite massif.

*Acknowledgements.* — This investigation was financially supported by the National Swedish Board for Energy Source Development. B. Collini, Department of Mineralogy and Petrology, University of Uppsala, M. Ambros and I. Lundström, The Geological Survey of Sweden, Uppsala, introduced us in the field and supported us with valuable information. The "divided bar" measurements were kindly carried out by Dr. Lachenbruch and coworkers, U.S.G.S., Menlo Park. Figures were drawn by Kerstin Berndtsson.

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