# Infiltration-conductivity determinations from non-steady-state infiltration tests

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Infiltration conductivity is a vital parameter in hydrogeological studies. In this paper, an in situ, falling-head method has been used, which allows of evaluation by stationary and non-steady-state theories. The results show that the non-steady-state theory is the most promising one, especially when using an evaluation technique according to ordinary well functions. The latter method, however, was difficult to treat without late-time data. Accordingly, the results are given of both steady and non-steady solutions. They show that infilling material containing hygroscopic components, such as bentonite, clearly reduces the conductivity to the order of  $10^{-11}$  m/s, compared with the original conductivity of about  $10^{-7}$  m/s before the wetting process had taken place.

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## Introduction

The infiltration conductivity of a soil is of importance, as it determines the rate of the groundwater recharge. The amount of precipitation which actually contributes to the recharge is determined either by this parameter or by the available precipitation (real precipitation minus evapotranspiration).

The infiltration conditions of soils are normally determined by using various kinds of infiltrometers. Double-ring infiltrometers were used by, for instance, von Brömssen (1968), Eriksson (1977), and Olsson (1979). In cases in which the soil surface was overflowed by water, another kind of infiltrometer was used by Wedel & Holmstrand (1976). Both these kinds of tests may be regarded as conductivity tests with constant head, and the infiltration rate is given by the water uptake of the soil after a certain period of saturation. Then normal rates obtained from this kind of test in till soils are about 1—10 mm/min, which corresponds to a conductivity value of about  $10^{-5}$  — $10^{-4}$  m/s.

In the present study, a different method has been used, employing what may be regarded as a falling-head, double-ring infiltrometer. As the term indicates, the method allows of measurements of the head transience being made during a test and thus it is possible to evaluate the results either as a conventional, constant-head test under assumed pseudo-stationary conditions or as a falling-head test under the actual, non-steady conditions which prevail on most recharge occasions.

Originally, the tests were carried out with the aim of testing different soil types and soil mixtures with respect to their suitability as tight, covering material to reduce the infiltration in wastestorage plants (Karlqvist 1981). A number of studies have been carried out on natural soils, as well as on these artificial soil types. Therefore, a large number of analyses exist, both of natural soils and of other redeposited materials, and a representative part of the total material is presented in this paper.

#### Infiltration tests

The equipment used for the infiltration tests is shown in Figure 1. The main parts are two concentric tubes with diameters of 67 mm and 150 mm respectively, each tube being about 750 mm in length. Both tubes are forced into the topsoil down to a approximate depth of 50—100 mm, which means that about 650—700 mm of the cylinders remain above the soil surface. The tubes are then filled with water up to the top of the inner tube, after which one starts by measuring the level in the inner tube at certain time intervals. The water level in the outer cylinder gives



Fig. 1. The measuring tubes used for the infiltration tests. Protograph by T. Olsson.

Table 1. Soil types tested with falling-head infiltrometer. N = natural soil, R = redeposited soil, C = soilcovered with a loose cover of waste material, and <math>U =uncovered soil.

Test no.	Soil type	Status	Admixture	
0 sample 1	Compacted waste			
1	material	U		
0-sample 2	Compacted waste			
-	material	U		
1	Sand	R/U	10 % bentonite	
2	Sand	R/C	10 % bentonite	
3	Clay	R/U		
4	Clay	R/C		
5	Till	R/C	5 % bentonite	
6	Till	R/U	5 % bentonite	
7	Till	R/C	5 % bentonite	
8	Till	R/U		
9	Till	R/C		
10	Till	Ν		
11	Clay	R*		
12**	Till	R/C	5 % bentonite	
13**	Till	R/C	5 % bentonite	

\* This clay was deposited in 1977 and the infiltration measurements were made 4 years later.

\*\* Only for use in the late-time analyses.

a flow approximating to that indicated in Figure 2, i.e. a parallel flow from the inner tube. Owing to this flow pattern, the fall in head in the outer cylinder will be faster than that in the inner one. In order to maintain the parallel flow, the head in both rings must be kept in balance.

All the measurements were repeated at at least two different places in each soil type, in order to



Fig. 2. Schematic view of the measurements with the flow pattern in the underlying ground.

reduce the effects of any inhomogeneities that might occur. The reason for this procedure is that the measuring tubes cover an area of only  $35 \text{ cm}^2$ . In order to study the effects of varying degrees of saturation, the measurements were repeated three times in sequence at every place. This made it possible to cover the trends in conductivity versus increasing saturation.

The tests included in this paper were all carried out on the soil types described in Table 1. On the testing occasions, the soils were all dried up after a period of some weeks without any rain.

### Flow theory

The set-up described above gives a volumetric flow which decreases in time with decreasing head, as indicated in Figure 3. Additionally, the infiltration conductivity will vary during the test, owing to varying degrees of saturation. This gives a flow which depends on a number of variables:

$$Q(\partial t) = F[(k, h, S)\partial t], \qquad (1)$$

where  $Q(\Im t) =$  volumetric flow, k = infiltration conductivity, h = water head, S = storage coefficient.

All the variables change with time. The conductivity at the beginning is the non-saturated conductivity, which increases with increasing saturation. The conductivity is also decreased by the swelling clay minerals included in the soil. The acting head of water decreases as the water infiltrates in to the ground. As the soil is not satu-







Fig. 4. Notations used in the measurements.

rated with water, a varying proportion of the total porosity has to be refilled with water before the conductivity governs the infiltration rate.

To evaluate the complex, functional relation in equation (1), it is necessary to make some approximations. Assuming constant conductivity, common evaluation techniques for non-steady solutions may be used, though somewhat modified. The tests may also be analysed, if they are regarded as pseudostationary and the values are adjusted with respect to the high head of water which governs the infiltration.

#### Stationary solution

Different approaches may be used in evaluating the tests as stationary ones; none, however, will reflect the actual conditions during the test. According to Darcy's law:

$$k = \frac{Q}{IA}, \qquad (2)$$

where k = hydraulic conductivity,

Q = volumetric flow,

I = hyraulic gradient,

A = a cross-sectional area of the flow.

At a specific time after the start of the test (cf. Figure 4 for the notations),

$$Q = \frac{A[h_2 - h_1]}{t_2 - t_1},$$
(3)

where  $h_1$  and  $h_2$  are the heights of the water column above the ground at the times  $t_1$  and  $t_2$ .



Fig. 5. The decrease in water head versus the logarithm of the time (Jacob plot).

The hydraulic gradient may be written:

$$I = \frac{H_0 + \frac{1}{2}(h_1 + h_2)}{H_0},$$
(4)

where 
$$H_0 =$$
 thickness of the tested soil layer or  
the depth where the flow pattern  
deviates (approximately four times  
the diameter of the inner tube).

If equations (2)—(4) are combined, the result will be:

$$\mathbf{k} = \frac{2\mathbf{H}_0 \,\Delta \mathbf{h}}{[2\mathbf{H}_0 + \mathbf{h}_1 + \mathbf{h}_2] \,\Delta \mathbf{t}} \,. \tag{5}$$

This relation will reach the real value of the conductivity asymptotically. In order to obtain a fairly accurate value, it is necessary to use late time data for the analysis.

#### Non-steady solution

On analysing the head transience in Figure 5 (cf. Figure 4), it can be seen that

$$\partial \mathbf{Q} = -\mathbf{A} \ \partial \mathbf{h}$$
 (6)

and

$$\partial Q = k \frac{h}{H_0} A \partial t.$$
 (7)

Combining these equations gives:

$$-\frac{\partial h}{h} = \frac{k}{H_0} \partial t, \qquad (8)$$

which, after integration and with proper boundary conditions, yields

$$k = \frac{H_0}{t_2 - t_1} \ln \frac{h_1}{h_2}, \qquad (9)$$

which is one of the non-steady solutions for the infiltration tests.

An approach based on Jacob's approximation has also been used, however modified compared to the original theory for well-analyses. This method is based on a derivation for a point source rather than a line-source. The relevant solution for a point source is as follows:

$$\Delta s = \frac{Q}{2\pi T d} \ erfc \left[ \frac{r^2 S}{4T t} \right], \tag{10}$$

where  $\triangle s =$  change in head, T = transmissivity, r = radius of the point source, S = storage coefficient, t = time, erfc(X) = complimentary error function of X.

This equation was originally derived for heat transport by Carslaw & Jaeger (1959), but is here modified for the present purpose. Equation (10) may be approximated to:

$$\Delta s = \frac{2.3 \,\mathrm{Q}}{2\pi \mathrm{Td}} \log \frac{2.25 \,\mathrm{Tt}}{\mathrm{Sr}^2}.$$
 (11)

This approximation is then similar to that of Jacob. The equation indicates that a plot of s versus log t should produce a straight line. This is also the case in the tests included in this study. The equation may be solved by the data plot and the following relation:

$$T = \frac{-0.366 \,\mathrm{Q}}{\triangle \mathrm{sd}}.\tag{12}$$

However, as both the head and the infiltration capacities vary with time, this relation has to be changed. Thus, assuming that k is constant and that

$$T = k \frac{d}{2} \sqrt{\pi}, \qquad (13)$$

it is then possible to determine that

$$\mathbf{k} = 0,325 \, \frac{\triangle \mathbf{h}}{\triangle \mathbf{s} \triangle \mathbf{t}} \,, \tag{14}$$

which may be used to determine the infiltration conductivity, as well as those methods mentioned above. For a complete test cycle it will be:

$$\mathbf{k} = 0,325 \frac{\mathbf{h}_0}{\triangle s[\mathbf{t}_e - \mathbf{t}_o]} \tag{15}$$

or

$$\mathbf{k} = \frac{0,325}{\bigtriangleup s} \cdot \frac{\mathbf{h}_0}{\mathbf{t}_0 \left[ 10^{\left(\frac{\mathbf{h}_0}{\bigtriangleup s}\right)} - 2 \right]}.$$
 (16)

The latter equation yields the conductivity after a long time of saturation with an unlimited amount of water available for the infiltration. This is of course of interest for those soils where swelling material is present which decreases the conductivity with time. However, in order to determine the infiltration capacity for natural conditions, the data obtained after a shorter period are more likely to reflect the actual infiltration conditions.

For the evaluation with the last method, two different approaches have therefore been made:

- \* Calculation with early-time data to obtain a conductivity value which reflects the actual infiltration conditions. The evaluation was made with early-time data, within one hour of saturation, i.e.  $\triangle t = t t_0 < 1$  hour;  $\triangle h < h_0$ . For those tests which were as conductive as to allow a full cycle to take place in shorter time, the complete cycle was used, i.e.  $\triangle t = t_e t_0$ ;  $\triangle h = h_0$ .
- \* Calculation with late-time data,  $\triangle t = t_e t_0$ ;  $\triangle h = h_0$ , in order to obtain any possible differences in time. This calculation was mainly made for low-conductive formations.

## Evaluation methods

For the evaluation of the test, three different methods were used:

- 1. Stationary solution according to equation (5).
- 2. Non-stationary solution according to equation (9).
- 3. Non-stationary solution according to equation (14) (modified Jacob approximation).

## Results and discussion

In all 15 different soil types have been tested, each at at least two points with three test cycles at each point. This gives a total number of more than 90 tests. The soil types used as impervious

Table 3. Late-time infiltration-conductivity values. = equal conductivity,  $\downarrow$  decreasing conductivity.

Test no.	Conductivity (10 <sup>-6</sup> m/s)	Trend	Admixture	
0-sample 1	8	Ļ		
0-sample 2	0,3	Ļ		
1	0,0005	i i i	10 % Bentonit	
2	0,0009	444	10 % Bentonit	
3	0,001	1 I		
4	0,08	=		
5	0,0004	$\uparrow$ $\uparrow$ $\uparrow$	5 % Bentonit	
6			5 % Bentonit	
7	0,02	11	5 % Bentonit	
8	0,1	=	, -	
9	2	=		
12	2	_	5 % Bentonit	
13	8	_	5 % Bentonit	

#### % Relative distribution



Fig. 6. Comparison between methods 1 and 2.

layers are either bare or covered with about 30 cm top soil to faciliate the revegetation. In this case the top soil was removed and the underlying soil was tested. The results are summarized in Table 2. As will be seen, methods 1 and 2 (stationary and first, non-stationary solution) agree very well. All the mean values are of the same order of magnitude and the relative imperviousnesses are about the same. The third method (Jacob's non-steady solution) gives results equivalent to those of the first methods. The Jacob approximation is based on the most accurate

model-description of the infiltration process and thus, its result should be the most reliable.

The conductivity ratio obtained between methods 1 and 2 is visualized in Figure 6, which shows that the non-steady solution always resulted in higher values. In no case did the stationary solution produce the higher value. It is also shown that no single measurement resulted in a conductivity for the non-steady solution which was more than nine times higher than the stationary solution. In 31 per cent of all tests, the ratio is as low as three times. This agreement is very good

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Test no.	Method 1, conductivity (10 <sup>-6</sup> m/s)	k <sub>0</sub> :k <sub>n</sub>	Method 2, conductivity (10-6 m/s)	$k_0:k_n$	Method 3, conductivity (10-6 m/s)	k <sub>0</sub> :k <sub>n</sub>
0-sample	400	1	2000	1	1000	1
1	9	45	20	84	80	12
2	30	16	50	40	20	50
3	20	24	30	59	300	3
4	< 0,1	999	<0,1	999	< 0,1	999
5	8	51	20	100	40	22
6	< 0.1	999	< 0.1	999	< 0.1	999
7	20	27	60	27	50	20
8	2	273	4	364	< 0.1	999
9	30	13	50	40	20	50
10	100	4	500	4	100	10
11	<0,1	999	<0,1	999	<0,1	999

Table 2. Infiltration-conductivity values of the different soil types. Relative values are given for all tests related to the 0-sample 1.



Fig. 7. Comparison between methods 2 and 3.

regarding the very inhomogeneous geological materials which were tested and the different approaches used.

Figure 7 shows a comparison between methods 2 and 3, and again, the agreement is good.

Additionally, the evaluation according to Jacob's approximation was used in order to obtain late-time conductivities, that is, after a long period of water saturation. This was done in order to indicate any possible effect of the infilling of bentonite in the different soil types. Bentonite is an altered volcanic ash with a high content of clay minerals mainly belonging to the montmorillonite group. The usefulness of bentonite for sealing purpose is due to its swelling properties.

The result of the determinations is shown in Table 3, which illustrates that the conductivity decreases with time, at most several powers for those soils in which bentonite was included. The conductivity also decreased in some other soil types, but generally to a smaller degree and then especially in clay and clayey till, which may in themselves contain swelling clay minerals. The



Fig. 8. The relation between infiltration conductivity and infiltration capacity.



Fig. 9. Values of the infiltration capacity for different soil types. 1. Present study, 2. von Brömssen 1968, 3. Bergman 1972, 4. Olsson 1980, 5. Flodqvist 1931, 6. Gustavsson 1946, 7. Sahlström 1911, 8. Danfors 1967, 9. Fleetwood 1965, 10. Tamm 1931, 11. Skoglund 1972, 12. Granström & Larsson 1970.

figures given in the table are not accurate, but they give a fair idea or the trends for the different materials.

The infiltration conductivity is related to the infiltration capacity (normally given in the literature), as indicated in Figure 8. As regards the infiltration-conductivity values given in Table 2, it will be seen that the capacity for the soils lies between 100 mm/min for the loose cover and less than 0,01 mm/min for bentonite mixed soils and clay. This is for the short-time data.

For late-time data, the soils mixed with bentonite become almost impervious (values of about 10-3 mm/day occur). This indicates that these soils may be regarded as impervious if the water content is high enough. After periods of draught, these soils may allow considerable amounts of water to infiltrate before they become saturated. These conditions may occur when the snowmelting starts in spring and especially after the summer season. Thus, much of the water may infiltrate on account of high conductivity at the very initial stage of the snow-melting period. This is especially pronounced at the end of the vegetation period in autumn, when the soil has been drained by evapotranspiration. During the summer, only small volumes may infiltrate. The infiltration should therefore clearly be reduced in these soil types, and this has in fact also been verified by Karlqvist (1981).

The above-mentioned values for the short-time data are of the same order of magnitude as those normally found in the literature for similar soil types, as shown in Figure 9.

## Conclusions

The methods used for determining the infiltration conductivity in this paper have all been proved to be valuable. However, some difficulties have occurred in the analysis of the tests, but it seems that the non-steady solution is the most reliable. Although, for soil types which include swelling materials, such as bentonite or other hygroscopic material, the extended Jacob's approximation gives the best guidance to the sealing effects.

The results obtained are, in general, those which were expected and are comparable with those in other studies in the field. Bull. Geol. Inst. Univ. Uppsala, N. S. 9 (1982)

REFERENCES

- Bergman, G. 1972: Bestämning av infiltrationskoefficienter för bergytor och perkolationsbanor i jordlager.
- 87 pp. Stockholms Univ., Kvartärgeol. Inst. Stockholm. von Brömssen, U. 1968: Grundvattenbildning i geologiskt olika terrängavsnitt. Försök till bestämning av infiltrationskoefficienter. Metod-Teknik-Analys 1968, 33-110. Stockholm.
- Carslaw, H. S. & Jaeger, J. C. 1959: Conduction of heat in solids. 510 pp. Oxford University Press. Oxford.
- Danfors, E.: Infiltrationsmätningar i Nynäshamn 1963 och Finsta 1967. Not published - ref. Lundberg (1974).
- Eriksson, E. 1977: Ground-water recharge and surface run-off, *Striae* 4, 11–12. Uppsala.
- Fleetwood, Å. 1965: Infiltrationsmätningar inom Bredakärrsbäckens nederbördsområde jämte discussion rörande infiltrationsinverkan på avrinningen. Kungl. Tekn. Högskolan, Mimeographed report. Stockholm.
- Flodqvist, H. 1931: Kulturtechnische Grundwasserforschungen. Sver. Geol. Unders. C 371. 317 pp. Stockholm.
- Granström, J. & Larsson, T. 1970: Översiktlig avloppsutredning för del av Sävar kommuns kustområde. Kungl. Tekn. Högskolan, Mimeographed report. Stockholm.

- Gustafsson, Y. 1946: Untersuchungen über die Strömungsverhältnisse in gedräntem Boden. Acta Agric. Suec. II:I. 154 pp. Stockholm.
- Karlqvist, L. 1981: Avslutning av avfallsupplag, del-
- rapport. In press. Lundberg, A. 1974: Infiltration och perkolation i det översta markskiktet. En sammanställning av utförda undersökningar i Norden. Int. Hydrol. Decade, No. 35. 50 pp. Stockholm.
- Olsson, T. 1979: Hydraulic properties and groundwater balance in a soil-rock aquifer system in the Juktan area, northern Sweden. Striae 12. 72 pp. Uppsala.
- Olsson, T. 1980: Ground-water-level fluctuations as a measure of the effective porosity and ground-water recharge. Sver. Geol. Unders. Rapp. och Medd. 21. 46 pp. Uppsala.
- Sahlström, K. E. 1911: Några försök angående jordarters permeabilitet i naturen. Sver. Geol. Unders. C 245. 56 pp. Stockholm.
- Skoglund, E. 1972: Några försök att bedöma det sekundära porsystemets betydelse vid vattenströmning i de översta marklagren. Kungl. Tekn. Högskolan, Mimeographed report. Stockholm.
- Tamm, O. 1931: Studier över jordmånstyper och deras förhållande till markens hydrologi i nordsvenska skogsterränger. Medd. Statens Skogsförsöksanst. 26:2. 193 pp. Stockholm.