A Fractal analog for distributed seismicity

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A fractal analog has been constructed in order to model the distribution of earthquakes in a region. The analog consists of a third order parallel and series network of elements with a fractal distribution of sizes. Each element is given a random strength based on a Weibull distribution. Strain is applied to the network until an element fails; this is the analog of an earthquake. When a failure occurs the stress on the failed network is redistributed on adjacent elements and the failed element is replaced with a new random strength. The redistribution of stress may lead to induced failures with no additional strain required. The failure of a second or third order element prior to the failure of a primary element is the analog of a foreshock; foreshocks occur prior to 19 % of the primary element failures. The failure of a second or third order element after the failure of a primary element is the analog of an aftershock; aftershocks occur after almost all the failures of primary elements. The distribution of failures of elements of various sizes is associated with a distribution of earthquakes of various magnitudes; the equivalent b-value is 1.05.

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Introduction

The simplest model for seismicity is to assume a uniform displacement on a single fault. Earthquakes with a specified displacement occur at regular intervals. In some cases this approach can be successful in predicting the occurrence of earthquakes on major faults. This is essentially the basis for the time-predictable recurrence model for large earthquakes (Shimazaki and Nakata, 1980; Thatcher, 1984) and the characteristic earthquake model (Schwartz and Coppersmith, 1984; Aki, 1984). There is strong observational evidence that the earthquakes on the San Andreas Fault near Parkfield repeat in a regular way (Bakun and McEvilly, 1984).

However, random aspects of seismicity have also received considerable attention. The asperity model (Lay and Kanamori, 1980, 1981; Ruff and Kanamori, 1980, 1983a, b; Kanamori, 1981; Lay et al., 1982) and the barrier model (Das and Aki, 1977; Aki, 1979, 1984) introduce the concept of a failure strength distribution on a fault. Andrews (1980, 1981) applied a random stress function over a brittle fault surface to build a stochastic model and a distribution function of the difference between tectonic stress and frictional stress was used by von Seggern (1980) in a similar approach. A stochastic model proposed by Kagan and Knopoff (1981) simulates seismicity as a branching process. They have generated a synthetic earthquake catalog with statistical properties similar to actual catalogs. Allègre et al. (1982) and Newman and Knopoff (1982, 1983) have used a renormalization approach to crack fusion to model an earthquake. Smalley et al. (1985) modeled a fault as an array of asperities with a prescribed statistical distribution of failure strengths and investigated the transition from stable to catastrophic failure by a renormalization group method.

The scale invarience of geological phenomena is one of the first concepts taught to a student of geology. It is pointed out that an object with a scale, i.e. a coin, a rock hammer, a person, must be included whenever a photograph of a geological feature is taken. Without the scale it is often impossible to determine whether the photograph covers 10 cm or 10 km. The concept of fractals was introduced by Mandelbrot (1967) to provide a quantitative measure of scale invarient phenomena. Noting that the length of a rocky coastline increased as the length of the measuring rod decreased according to a power law, Mandelbrot associated the power with a fractal dimension.

The basic definition of a fractal distribution is

$$N = \frac{C}{r^D} \qquad (1)$$

where N is the number of objects with a characteristic linear dimension greater than r, C is a constant of proportionality, and D is the fractal dimension. As a mathematicl representation (1) could be valid over an infinite range; however, for any application there will be upper and lower limits of applicability. The essential feature of the fractal distribution is scale invariance, no characteristic length enters a power law distribution. The fractal distribution is applicable to a variety of geological phenomena including the size distribution of islands, fragmentation (where it is known as Rosen's law), and to seismicity.

Under many circumstances the number of earthquakes N with a surface-wave, magnitude greater than m satisfies the empirical relation (Gutenberg and Richter, 1954)

$$\log N = -bm + a \qquad (2)$$

where a and b are constants. The b-value is widely used as a measure of regional seismicity. Aki (1981) showed that (2) is equivalent to the definition of a fractal distribution.

The moment M of an earthquake is defined by

$$M = \mu \delta A \qquad (3)$$

where μ is the shear modulus, A the area of the fault break, and δ is the mean displacement on the fault break. The moment of an earthquake can be related to its magnitude by (Kasahara, 1981, p. 133)

$$\log M = cm + d \qquad (4)$$

where c and d are constants. Kanamori and Anderson (1975) have established a theoretical base for taking c = 3/2. These authors have also shown that it is a good approximation to take

$$M = \alpha r^3 \qquad (5)$$

where $r = A^{1/2}$ is the linear dimension of the fault break. Combining (2), (4), and (5) gives

$$\log N = -2b \log r + \beta \qquad (6)$$

where

$$\beta = 3/2\, bd + a - 3/2\, b \log \alpha \qquad (7)$$

and (6) can be rewritten as

$$N = \beta r^{-2b} \qquad (8)$$

A comparison with the definition of a fractal distribution (1) gives

$$D = 2b \qquad (9)$$

Thus the fractal dimension of seismic activity is simply twice the b value, since the b-value is usually in the range $0.8 < b < 1.2$ the range of fractal dimensions is $1.6 < D < 2.4$ (Hanks, 1979).

A fractal distribution of seismicity in a region can be interpreted in one of two ways, or any combination. The first is a fractal distribution of earthquake breaks on a single fault. The second is a fractal distribution of fault sizes with repetitive earthquakes on each fault. The second model has been proposed by Wesnousky et al. (1983) and provides the basis for the model considered in this paper. Fractal distributions of faults and their relationship to the fractal distribution of seismology have also been considered by King (1983) and Turcotte (1986).

In this paper, we model the seismic activity on a fault system by a fractal-based mechanical analog. We synthesize earthquakes on two time scales. The first is the tectonic time scale which is characterized by the recurrence time of major events on a fault system (decades to centuries or longer) and the second is the duration of a complete earthquake sequence (days to months). We obtain synthetic earthquake catalogs which include complete earthquake sequences with foreshocks, mainshock, and aftershocks as well as the recurrence of such sequences. The statistics obtained from these synthetic catalogs are compared with observations.

Our model has similarities and differences with previous laboratory and numerical models (Burridge and Knopoff, 1967; Dieterich, 1972; Cao and Aki, 1984). We model the transfer of stresses between faults in a manner similar to spring-mass systems but utilize a statistical distribution of asperity strengths rather than a friction model. We consider a model that includes both the spatial and the temporal dependencce of seismicity.

The simulation approach proposed in this paper differs from previous studies in the following ways: (1) We model a seismically active region as a group of sub-parallel fault planes with a self-similar size distribution instead of considering the stochastic properties associated with a single fault plane. (2) We build a model that is based on physical concepts, i.e. our model is not purely stochastic. (3) The concepts of remote loading and stress transfer are applied to distinguish seismic time scales. (4) Since we are not going to model the rupture process of each single earthquake in detail, the random failure strengths are assigned to each fault as a constant over the fault surface.

Analog seismic model

Our geometrically self-similar model is illustrated in Figure 1. We utilize a series of linear elastic elements (vertical bars) to represent a fault system. First, second, and third order elements are included in our model. The two first order elements are connected in series. The eight second order elements have one-quarter the cross sectional area of the first order elements and one-half the length. The thirty-two third order elements have one-quarter the cross sectional area of the second order elements and one-half the length. Using the definition of a discrete fractal set

$$N_i - r_i^{-D} \qquad (10)$$

where N_i is the number of elements of size r_i we find $D = 2$ for our mechanical analog.

The basic pattern of the model is two multi-element cells connected in series. Each of the cells includes five elements of two different orders with one higher order element in the center and two series-connected lower order elements on each side. Since the structure of the model is self-similar, we can add an additional order by replacing each of the lower order elements with another 5 element cell. Repeating this substitution, the order of this model can grow to infinity.

The entire system is placed under a tensional load as indicated by the arrows in Figure 1. This tensional stress represents the tectonic stress field that operates on a region that includes a fractal distribution of faults. The cross sectional area of each element is associated with the area of a fault.

Each element of the system is assigned a failure strength σ_f. We assume that the statistical distribution of failure strengths is given by a second order Weibull distribution

$$\Pr\left\{\frac{\sigma_f}{\sigma_0}\right\} = 1 - \exp\left[-\left\{\frac{\sigma_f}{\sigma_0}\right\}^2\right] \qquad (11)$$

where σ_0 is a reference failure stress. The failure strength of each element is determined using (11) and a pseudo-random number generator.

The tensional load on the system is increased until the stress on an element reaches its failure stress. The failure of this element is taken to be the equivalent of an earthquake. The magnitude of the equivalent earthquake depends upon the order (size) of the element that fails and the stress on the element at failure. After failure the stress (load) on the failed element is redistributed among the unfailed elements. This is equivalent to what happens after an earthquake. Some fraction of the regional stress will be transferred to adjacent faults. At the same time the failed element and the adjacent series element (which also now has zero load) are both replaced with new elements with random failure strengths and zero stress. Again this is equivalent to what happens on an actual fault. After an earthquake on a fault the distribution of asperities is almost certainly different and the strength of the fault may be increased or decreased.

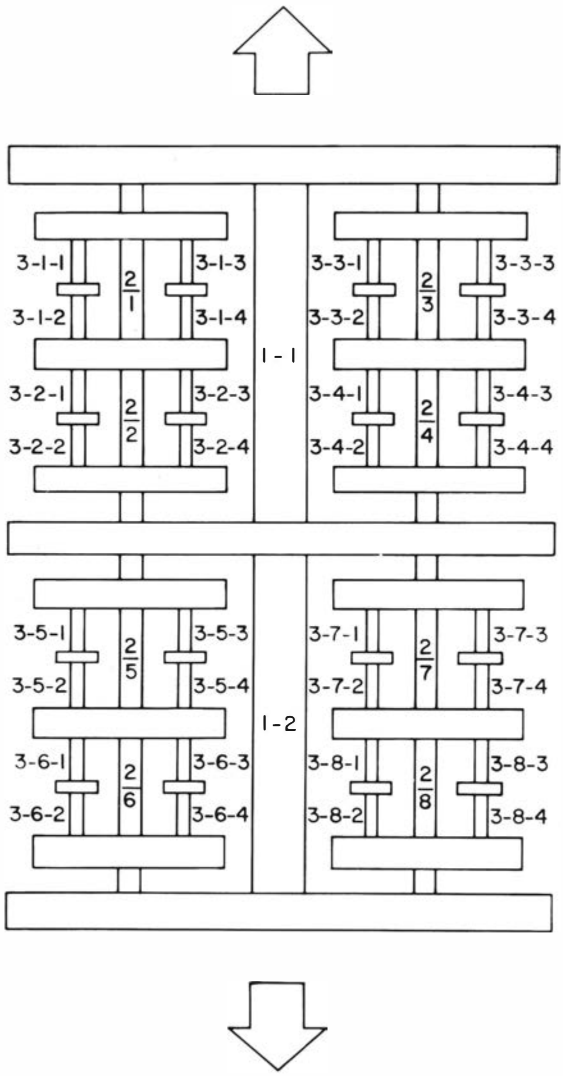

Figure 1. Illustration of the fractal-based, mechanical analog model.

After stress redistribution and element replacement, one or more other elements may have stresses above their failure stresses and thus will fail. If this occurs the stress redistribution and element replacement process is repeated. If no element fails after stress redistribution the tensional load is again increased until an element fails. The process can be continued indefinitely.

A set of failures without additional load is associated with foreshocks and aftershocks. Multiple shocks can also occur. Stress redistribution is an essential part of our model and is clearly responsible for the occurrence of the foreshocks and aftershocks. It should be noted that a failure in the upper half of our model will result in an increased strain in that half and a reduction in the lower half. The stress will increase on all unfailed elements in the upper half (except the adjacent series element) and will be reduced on all elements in the lower half.

Synthetic earthquake catalog

Carrying out the process described above a number of synthetic earthquake catalogs have been constructed. For computational convenience the length

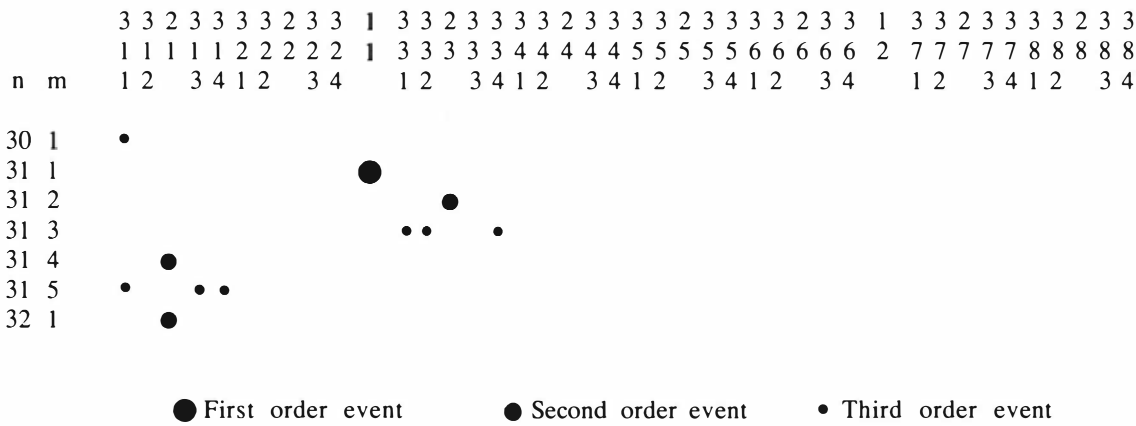

Figure 2. Illustration of a typical earthquake sequence. (a) above: Tabulation of element failures. (b) below: Tabulation of stress and failure stress for each element.

STRESS

n	m	3-1-1	3-1-2	2-1	3-1-3	3-1-4	1-1	3-3-1	3-3-2	2-3	3-3-3	3-3-4
30	1	0.4241	0.4241	0.4956	0.3391	0.3391	0.5458	0.0199	0.0199	0.4616	0.3994	0.3994
31	1	0.0009	0.0009	0.5374	0.3809	0.3809	0.5510	0.0251	0.0251	0.4668	0.4047	0.4047
31	2	0.2213	0.2213	0.7579	0.6013	0.6013	0.0000	0.2456	0.2456	0.6872	0.6251	0.6251
31	3	0.2613	0.2613	0.7979	0.6413	0.6413	0.0400	0.6192	0.6192	0.0000	0.9987	0.9987
31	4	0.2798	0.2798	0.8164	0.6598	0.6598	0.0586	0.0000	0.0000	0.1729	0.0000	0.0000
31	5	0.7237	0.7237	0.0000	1.1037	1.1037	0.1061	0.0476	0.0476	0.2205	0.0476	0.0476
32	1	0.0482	0.0482	0.2435	0.0482	0.0482	0.1752	0.1167	0.1167	0.2896	0.1167	0.1167

FAILURE STRENGTH

n	m	3-1-1	3-1-2	2-1	3-1-3	3-1-4	1-1	3-3-1	3-3-2	2-3	3-3-3	3-3-4
30	1	0.4240	1.4481	0.7846	0.7509	0.6896	0.5510	0.5694	0.3732	0.5085	1.4786	0.9455
31	1	0.3264	0.8797	0.7846	0.7509	0.6896	0.5510	0.5694	0.3732	0.5085	1.4786	0.9455
31	2	0.3264	0.8797	0.7846	0.7509	0.6896	0.9936	0.5694	0.3732	0.5085	1.4786	0.9455
31	3	0.3264	0.8797	0.7846	0.7509	0.6896	0.9936	0.5694	0.3732	0.5085	1.4786	0.9455
31	4	0.3264	0.8797	0.7846	0.7509	0.6896	0.9936	1.4597	0.4887	0.7023	0.4124	1.5208
31	5	0.3264	0.8797	0.2435	0.7509	0.6896	0.9936	1.4597	0.4887	0.7023	0.4124	1.5208
32	1	1.4475	0.5737	0.2435	0.9730	0.9364	0.9936	1.4597	0.4887	0.7023	0.4124	1.5208

of each catalog is limited to a sequence of about 250 earthquake sequences.

A typical synthetic earthquake sequence is shown in Figure 2. The number n is the number of times that additional load has been applied; this is equivalent to the number of tectonic time steps. The number m is the number of times that stress has been redistributed. Each element is numbered according to the system introduced in Figure 1. The occurrence of failures is illustrated in Figure 2a, the stress on each element σ/σ_0 is given in Figure 2b together with the failure stress on each element σ_f/σ_0.

The increase in stress between steps 30 and 31 induces a mainshock on a first order element (1−1). It is seen that the stress on this element equals 0.5510 the failure stress. This mainshock is followed by a cluster of eight aftershocks associated with four redistributions of stress. After the first redistribution of stress the stress on a second order element (2-3) 0.6872 exceeds the failure stress on this element 0.5085. After the second redistribution of stress the stresses on three third elements (3-3-1, 3-3-2, 3-3-4) exceed their failure stresses and three aftershocks occur. A third redistribution of stress results in the failure of a second order element (2-1). A fourth redistribution of stress results in the failure of three third order elements (3-1-1, 3-1-3, 3-1-4). A fifth redistribution of stress does not result in the failure of any elements so that the applied stress is increased until the second order element 2-1 fails.

Synthetic earthquake sequences with other features also appeared in the catalogs; two of these are given in Figure 3. The sequence is initiated with a second order foreshock on element 2-4 and this is followed by a third order foreshock on element 3-1-3. The mainshock then occurs on element 1-1. This is followed by two second order aftershocks (2-1, 2-4) and seven third order aftershocks (3-1-2(2), 3-1-3, 3-1-4, 3-4-1, 3-4-3, 3-4-4).

A sequence with a double main event is also illustrated in Figure 3. The sequence is initiated with a

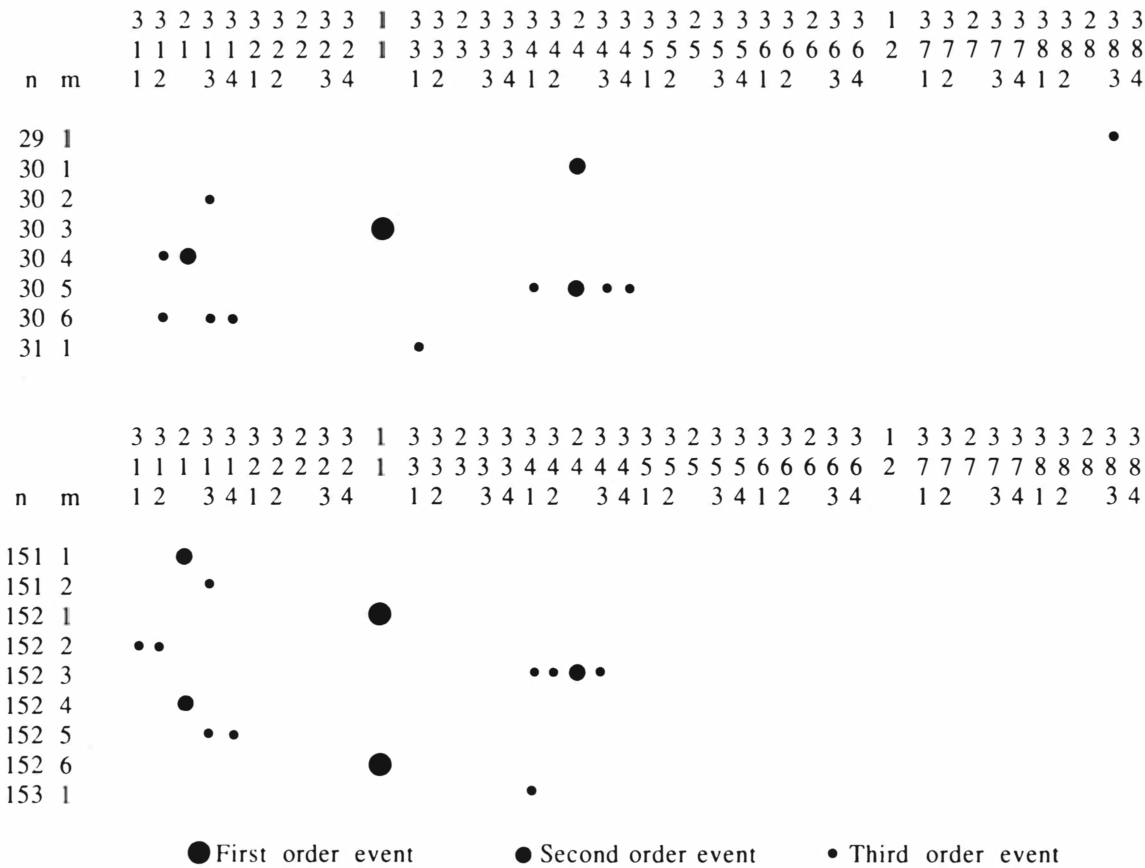

Figure 3. Tabulation of element failures for two typical earth sequences illustrating foreshocks (#30) and a double shock (#152).

main event on element 1-1. This is followed by two second order aftershocks (2-4, 2-1) and seven third order aftershocks (3-1-1, 3-1-2, 3-4-2, 3-4-2, 3-4-3, 3-1-3, 3-1-4). And the sequence terminates with a second main event on element 1-1.

In the synthetic catalogs that we have generated, 19 % of the main events had foreshocks. This is in excellent agreement with the study of actual earthquakes by von Seggern et al. (1981) who found 21 % of the earthquakes studied had foreshocks. Jones and Molnar (1976) studied only large shallow earthquakes that could be recorded teleseismically and found that 44 % had foreshocks. The occurrence of foreshocks before a fraction of the main events is a natural consequence of our stochastic model. In some cases the transfer of stress from a secondary element (fault) can trigger the failure of the primary element (fault). It is difficult to envision a non-stochastic model exhibiting this behavior.

Frequency-magnitude statistics

The synthetic earthquake catalogs that we have generated can also be used to determine frequency-magnitude statistics. For each failure of an element we take the failure stress σ_f to be the stress drop associated with the equivalent earthquake, this is always given by (11) in terms of the reference stress σ_o. The area A and length l of the failed element are referenced to the area A_o and length l_o of a primary element. The moment M of the syntthetic earthquake is given by (3) and the mean displacement is approximated by

$$\delta = \frac{\sigma_f l}{\mu} \qquad (12)$$

we assume that the magnitude is related to the moment by (4) with c = 1.5 and d = 16. From (3), (4), and (12) we have

$$m = \frac{1}{1.5} \log \left[A_0 \, l_0 \, \sigma_0 \left\{ \frac{\Delta\sigma}{\sigma_0} \right\} \left\{ \frac{1}{l_0} \right\} \left\{ \frac{A}{A_0} \right\} \right] - 10.7 \qquad (13)$$

In order to generate synthetic magnitudes we take $A_0 \, l_0 \, \sigma_0 = 10^{25}$ with the result that the synthetic earthquakes fall in a range of magnitudes between m = 5 to m = 7.5.

The cumulative frequency-magnitude distribution is given in Figure 4. A least square linear fit gives a b-value of 1.05. This is close to the values for actual earthquake catalogs that typically have b-values between 0.80 and 1.20. From (9) the corresponding

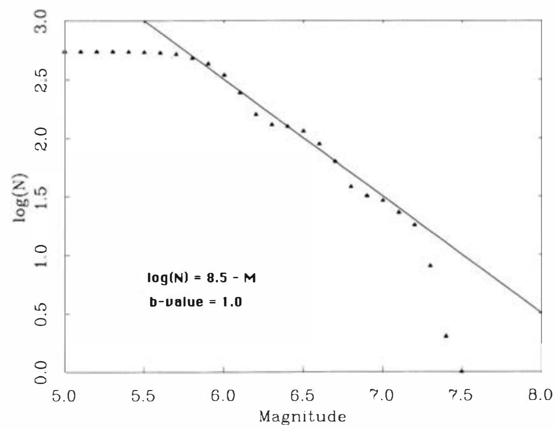

Figure 4. Frequency-magnitude statistics.

fractal dimension is D = 2.10. The fractal dimension for the earthquake distribution is very nearly equal to the fractal dimension D = 2 for the number-size relation for the elements (faults) in our model. The fractal behavior in Figure 4 is over a limited range of magnitude. This is not surprising since only three sizes of faults are considered in our model. If we extended our model to more orders we would expect that the range of applicability of the fractal distribution would increase.

Conclusions

In modelling earthquakes an essential question is whether the problem is deterministic or stochastic. In the sense that Schrodinger's equation is applicable, all problems are stochastic. However, continuum equations are now known to yield chaotic solutions. Lorenz (1963) derived a set of total differential equations that approximated the full equations for thermal convection. Numerical solutions of these equations exhibited chaotic behavior. That is, infinitesmal changes in initial conditions led to order-one variations in the evolving solutions. Many other solutions of this type have subsequently been found and their general behavior is classified as strange attractors; these solutions exhibit fractal behavior. It is now generally accepted that turbulence is chaotic, stochastic, and not deterministic. Although it is not possible at this time to fully specify the continuum equations governing the behavior of a seismogenic zone, it is very likely that they will generate chaotic solutions.

Stochastic problems that are scale invariant often yield fractal distributions. Fractal statistics are applicable to a variety of problems in turbulence. The fact that the frequency-magnitude relation for regional seismicity is a fractal is evidence that seismicity is stochastic. We have proposed a fractal-based model for seismicity. Earthquakes are modelled as the failure of mechanical elements. The elements have a fractal relationship between number and size. The elements are given random strengths. This is an analog to asperities with random strengths. Transfer of stress between elements is an essential feature of our model. Sets of earthquakes with foreshocks and aftershocks are modelled successfully. The frequency-magnitude statistics correspond to a b-value near unity which is in agreement with observations.

Alternative models can certainly be constructed. However, we feel that a fractal-based model is essential. The number-size relation of our elements is realistic and gives failure statistics that agree with earthquake distributions. Alternatives to the second -order Weibull distribution of strengths can be considered. Also the model can be extended to more orders of elements. We are quite encouraged that our relatively simple model can generate quite reasonable earthquake catalogs.

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